Higgs Branch RG Flows
--Decay and Fission of Magnetic Quivers

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Based on [2312.05304] + [2401.08757]
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Introduction

Supersymmetric gauge theories + Superconformal Field theories (SCFTs) with 8 Supercharges

$$3d\ \mathcal{N}=4 \qquad 4d\ \mathcal{N}=2 \qquad 5d\ \mathcal{N}=1 \qquad 6d\ \mathcal{N}=(1,0)$$

- Fields contained in supermultiplets:
  - Hypermultiplets $(\text{scalars}(\psi),\ \text{fermions})$
  - Vector multiplets $(\text{gauge field},\ \text{scalars}(\phi),\ \text{fermions})$

- Rich vacuum structure → Study moduli space of vacua
  - Higgs Branch (Paramterized by ψ)
  - Coulomb Branch (Paramterized by ϕ)

Higgsing (Higgs branch RG flows)

- Def: Higgsing (along the Higgs branch)

- Give VEV to scalars in hypers ψ
  - →Gauge bosons become massive
    - → Gauge symmetry breaks into a subgroup

Familiar Higgs mechanism!
E.g. Electroweak: $SU(2) \times U(1) \to U(1)$

Higgsing

Goal:

Given a theory $\mathcal{T}$, find all the daughter theories $\mathcal{T}'$ it Higgses to and the granddaughter theories $\mathcal{T}''$ etc.

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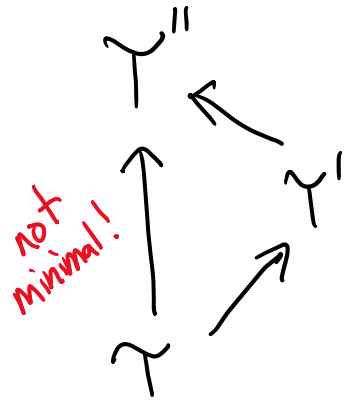
Minimally



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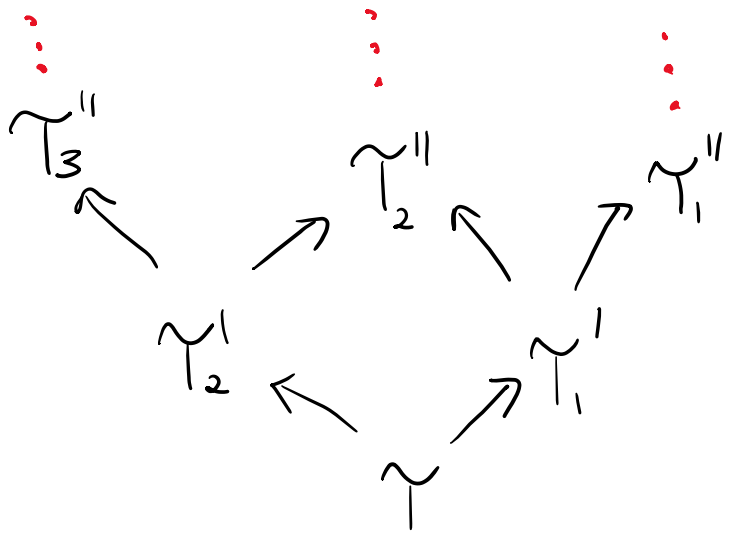
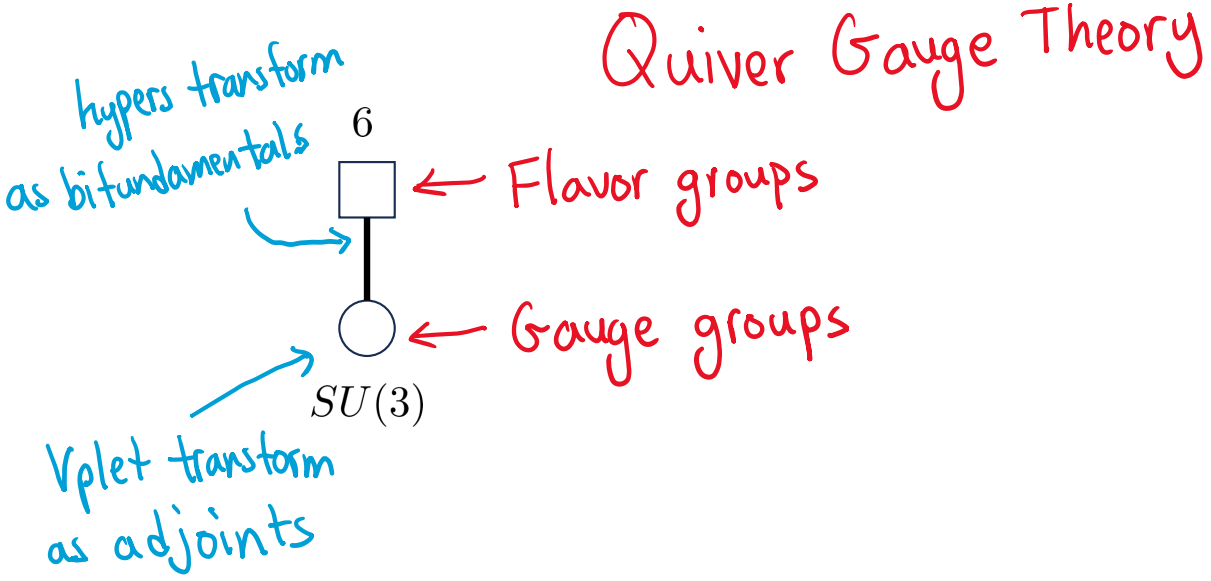


Put everything into a phase diagram (also called Hasse diagram)

Example

- SU(3) gauge theory with 6 flavors



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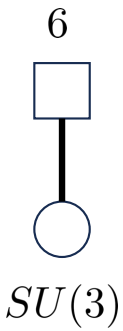


Higgs branch can be found by looking at F-terms and D-terms

Practically through the HyperKähler quotient

Higgsing

How?
Group theoretic methods (old)

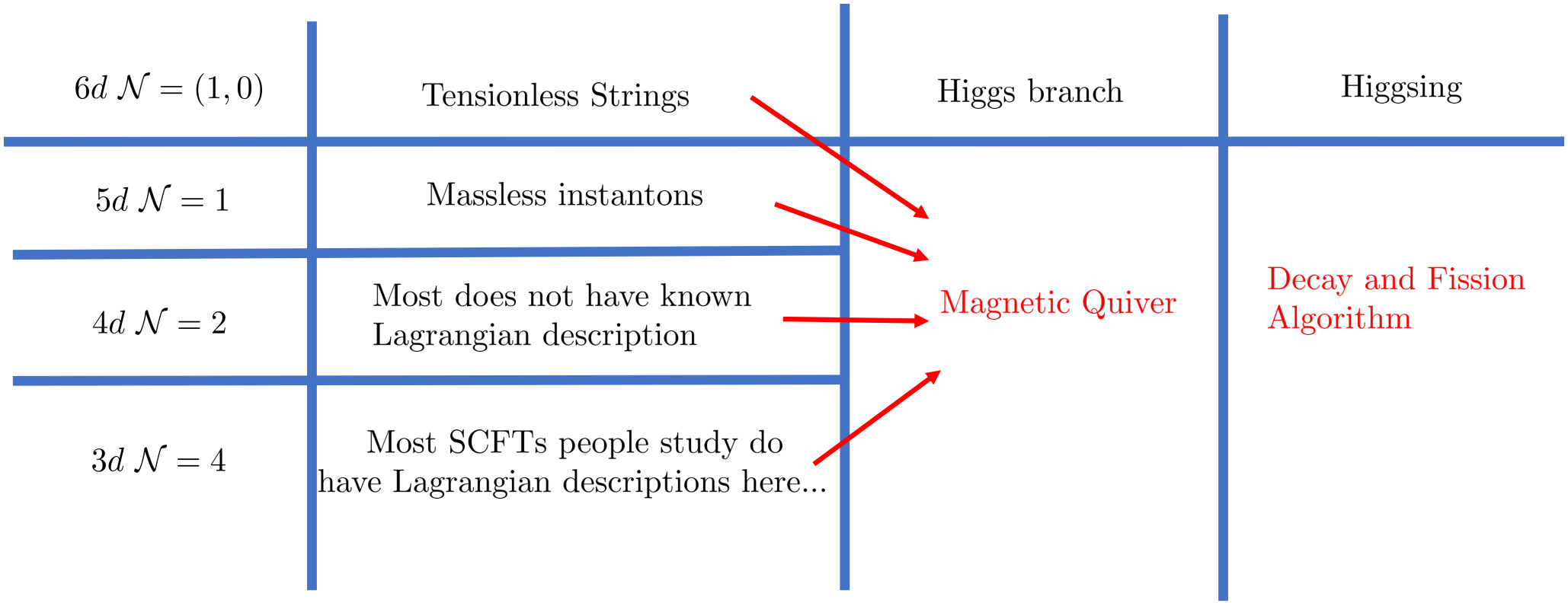
Alas. Things are not so simple

Traditional Methods

Higgs branch $\rightarrow$ HyperKahler Quotient
Higgsing $\rightarrow$ Group theoretic methods

Requires a **Lagrangian** Description

Lack of Lagrangian amongst SCFTs



Magnetic Quiver

Given a theory $\mathcal{T}$ in dimensions $d = 3, 4, 5, 6$, the corresponding Magnetic Quiver (MQ) is a $3d$ $\mathcal{N} = 4$ theory such that:

[Wenbin's talks]

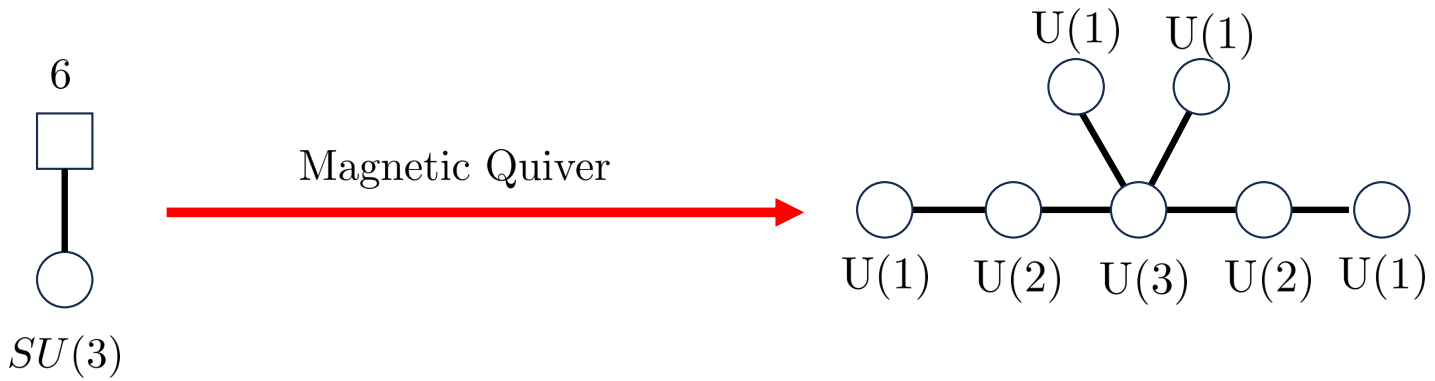
$$\text{Higgs}^{d=3,4,5,6}(\mathcal{T}) = \text{Coulomb}^{3d\ \mathcal{N}=4}(\text{MQ})$$

How to get Magnetic Quivers?

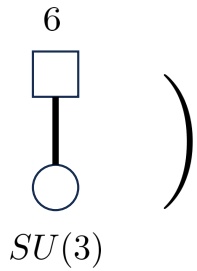
Find brane system that describes its Higgs branch

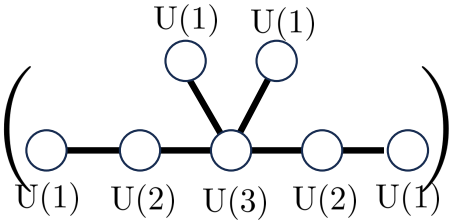
$6d\ \mathcal{N} = (1,0)$	IIA Branes: D6-D8-NS5-O8 system	Magnetic Quiver Extraction Algorithm
$5d\ \mathcal{N} = 1$	Brane webs: (p,q)5-branes-[p,q]7-branes system	
$4d\ \mathcal{N} = 2$	Bottom-Up construction + "Folding Quivers"	
$3d\ \mathcal{N} = 4$	IIB: D3-D5-NS5 brane systems	

$4d\ \mathcal{N}=2$ $3d\ \mathcal{N}=4$



$4d\ \mathcal{N}=2$ $3d\ \mathcal{N}=4$

$\text{Higgs}^{4d\ \mathcal{N}=2}$ 

$= \text{Coulomb}^{3d\ \mathcal{N}=4}$ 

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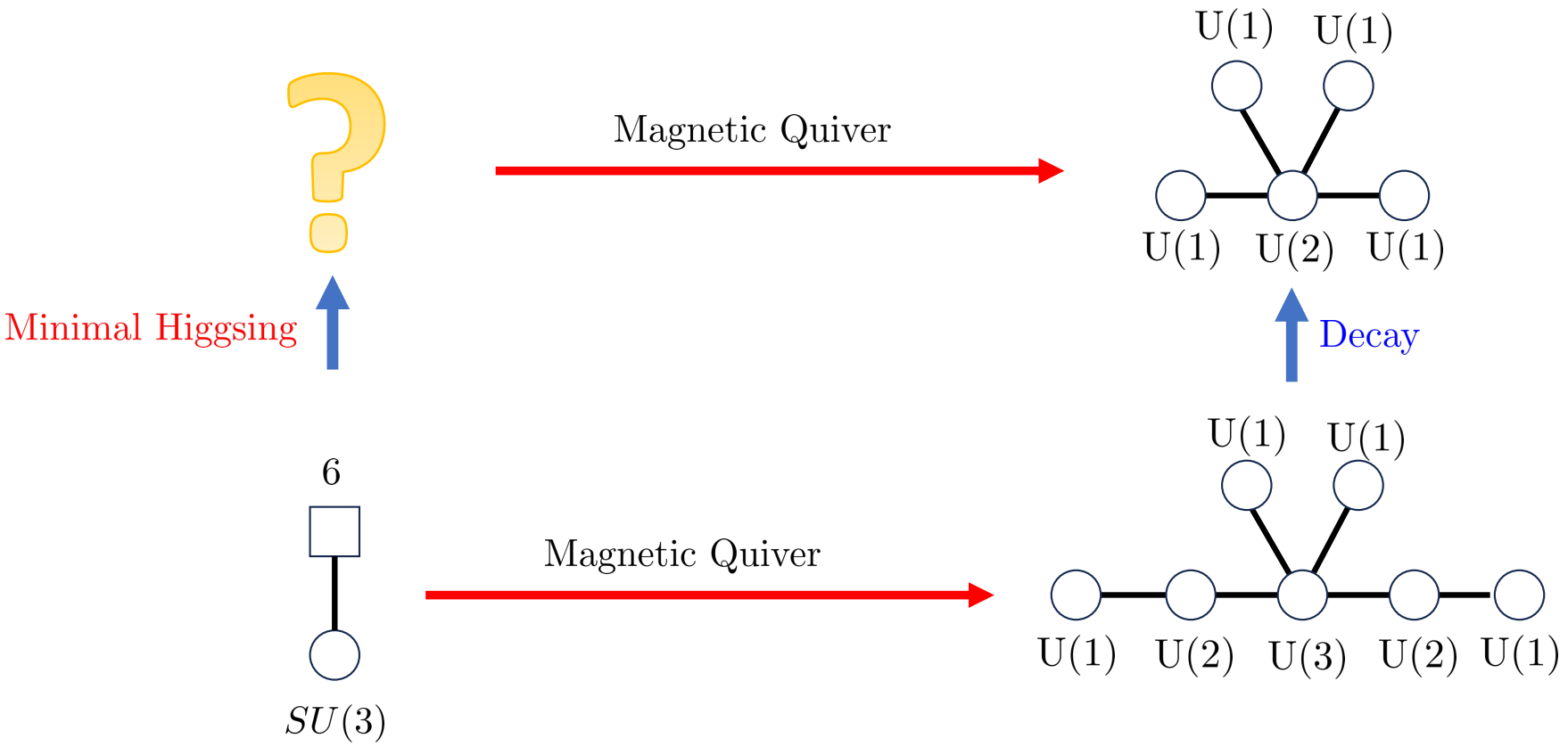
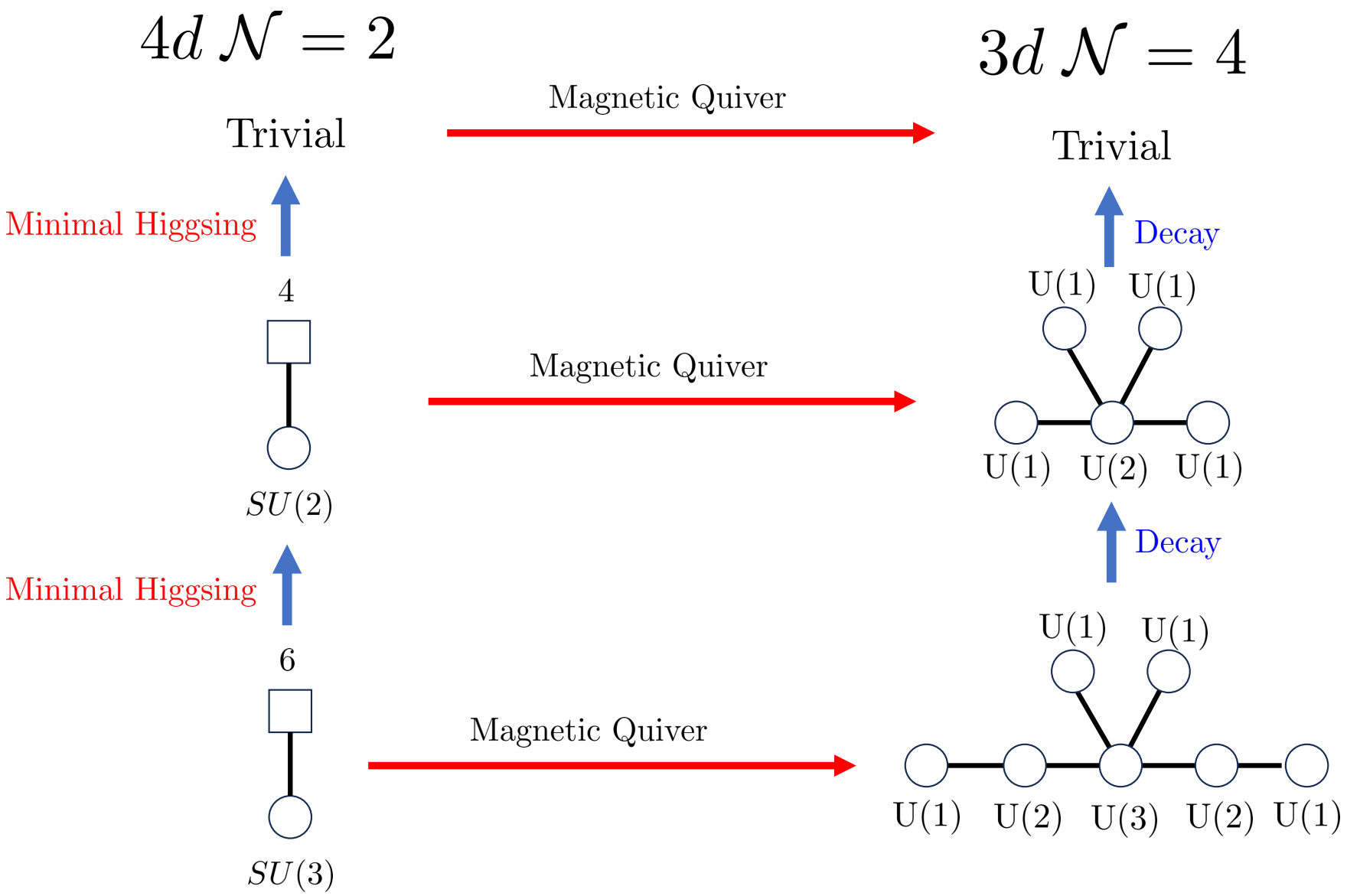
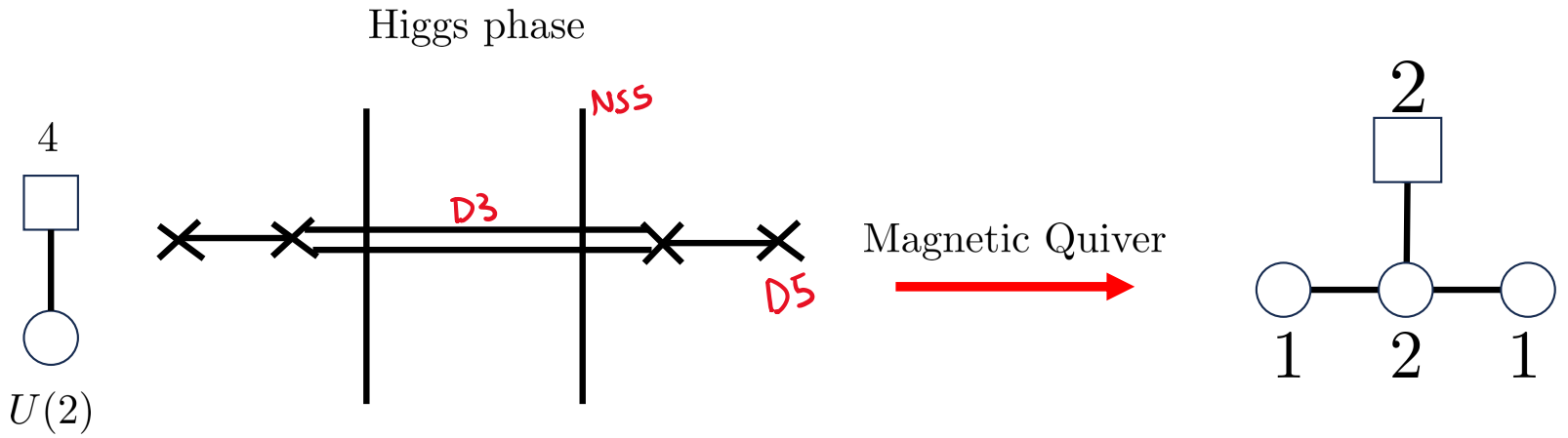



$4d\ \mathcal{N}=2$
$3d\ \mathcal{N}=4$
Magnetic Quiver
Trivial
Trivial
Minimal Higgsing
Decay
4
U(1) U(1)
Magnetic Quiver
U(1) U(2) U(1)
$SU(2)$
Minimal Higgsing
Decay
6
U(1) U(1)
Magnetic Quiver
U(1) U(2) U(3) U(2) U(1)
$SU(3)$

A new Higgsing Algorithm:

Decay and Fission of Magnetic Quivers

How does Higgsing work?



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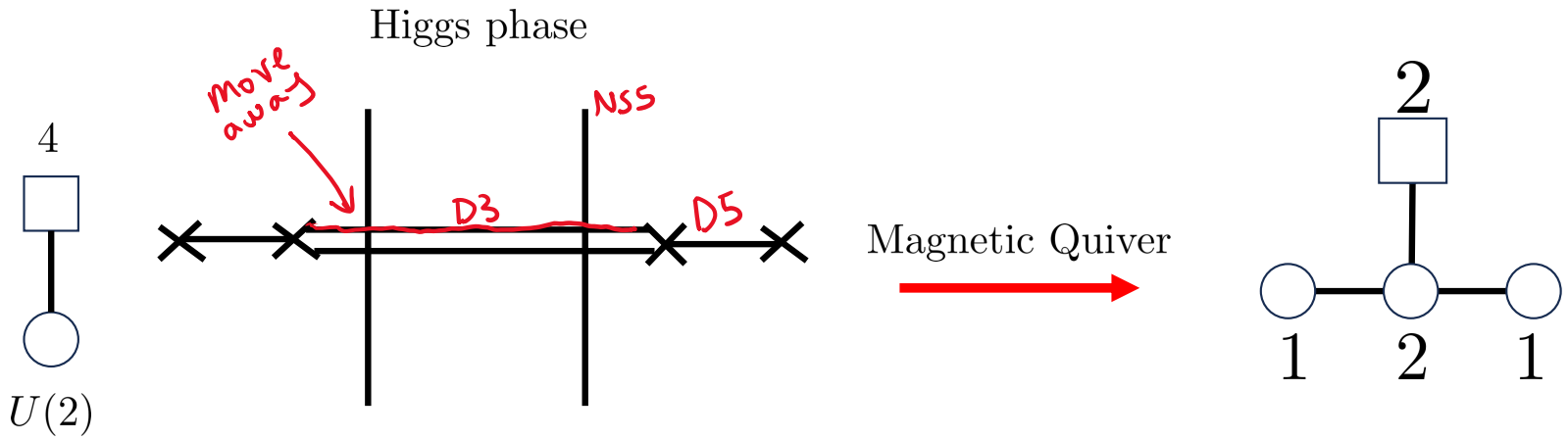

Moving the D3 brane away along the D5 branes is a Higgsing

$3d\ \mathcal{N} = 4$

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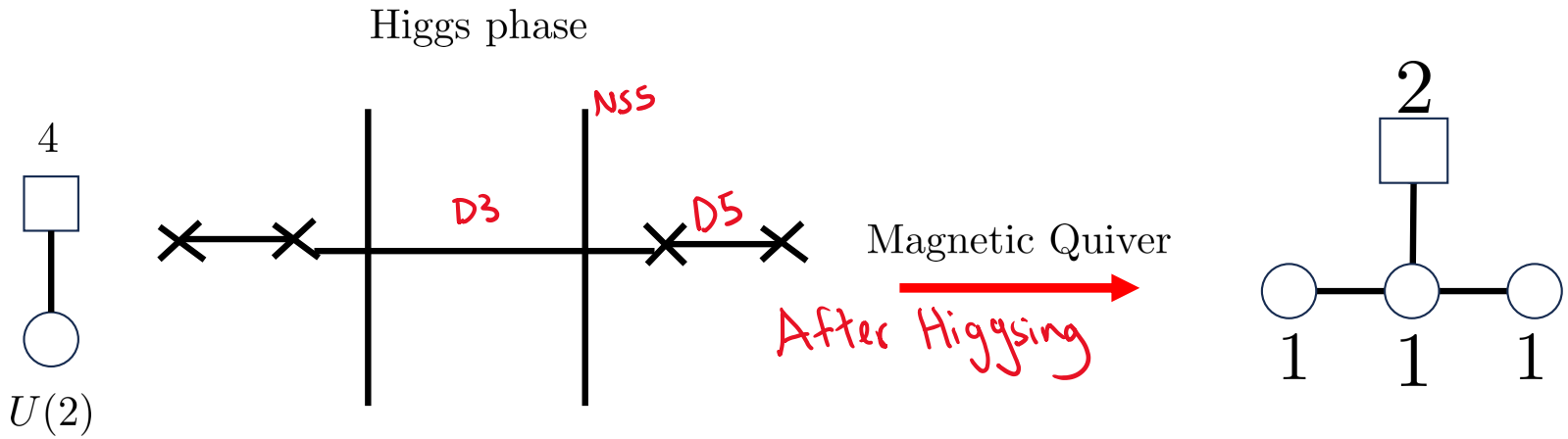

Moving D3 brane corresponding to reducing the rank of the $U(2)$ gauge group in the magnetic quiver by one

$3d\ \mathcal{N} = 4$

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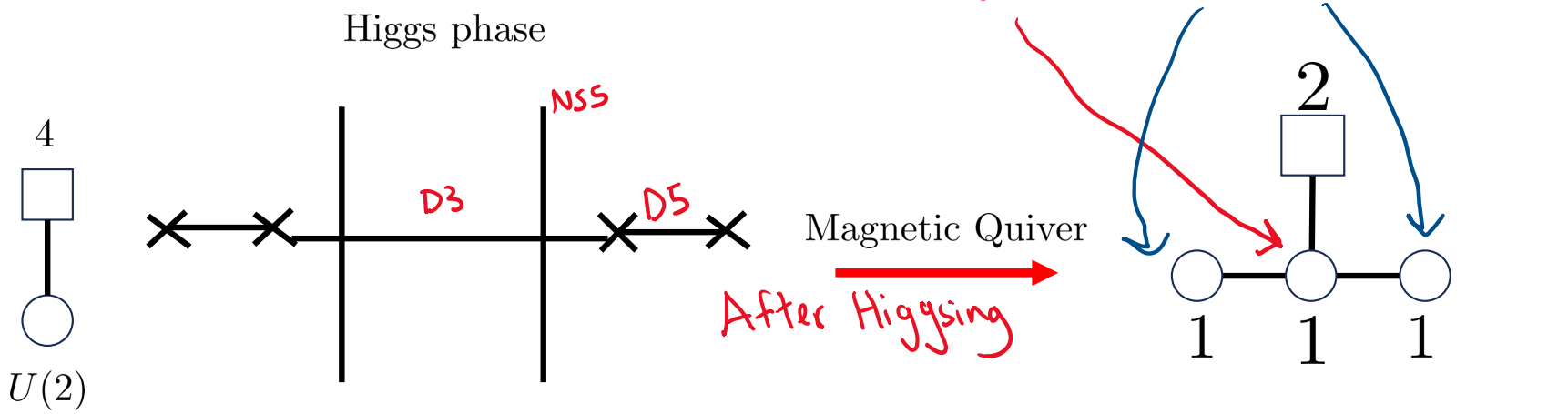
How does Higgsing work?

balance (b) = #edges from neighbors
~2×rank

b≥0 (good) b=-1 (ugly) b<-1 (bad)



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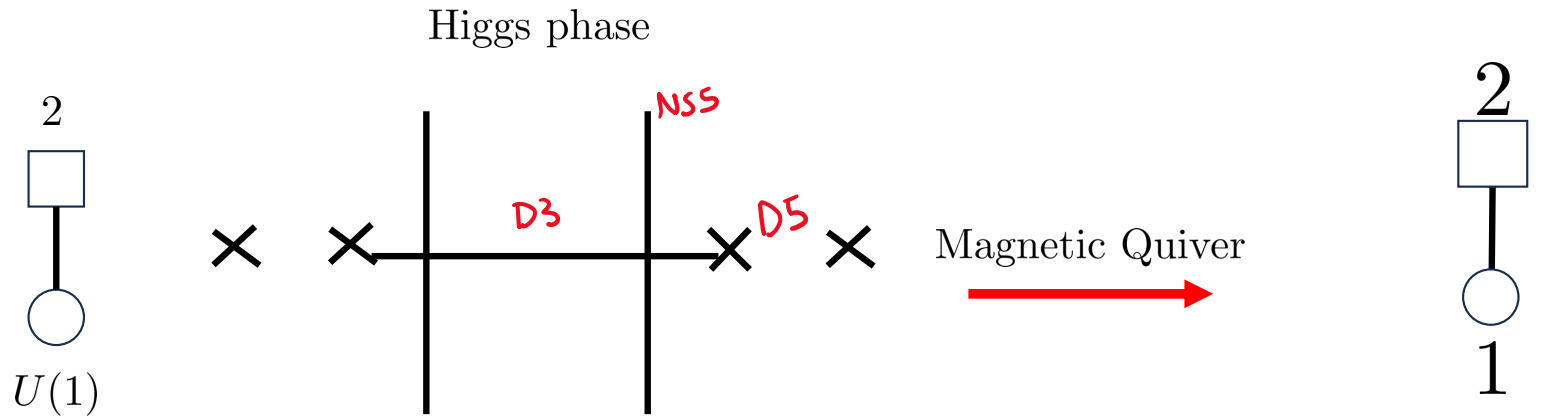

Higgsed Theory

Resulting quiver contains free hypers which we remove doing Seiberg duality movies

The resulting magnetic quiver correctly corresponds to the Higgsed theory of $U(1)$ with 2 flavors

Decay

Given a Magnetic Quiver, find all subquivers such that each gauge node is good

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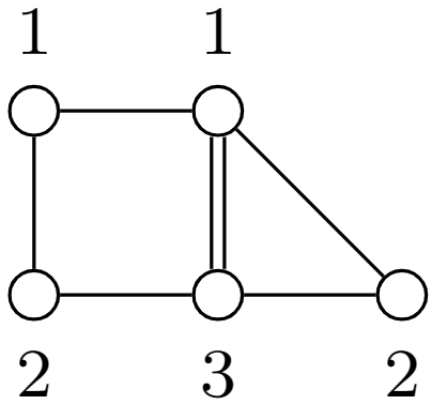

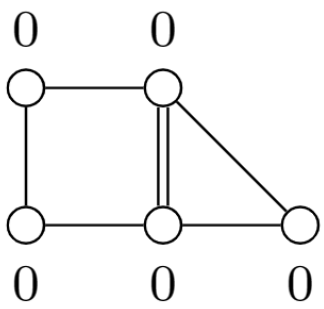
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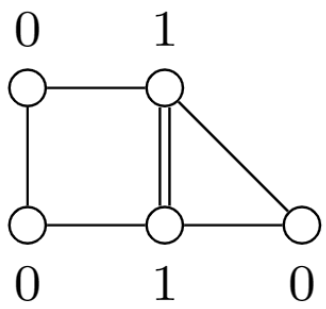
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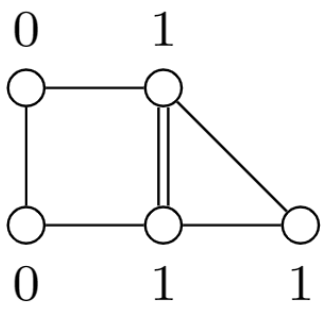
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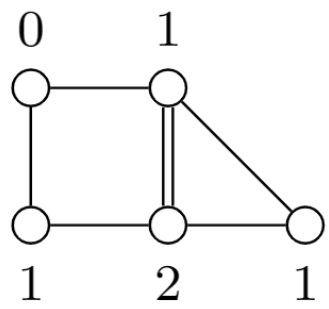
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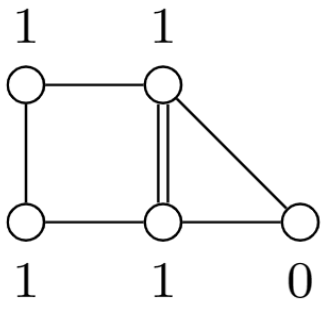
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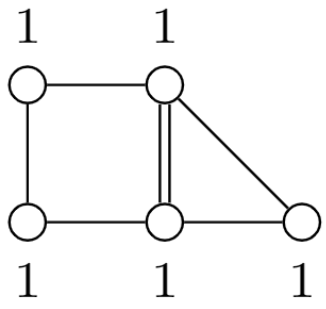
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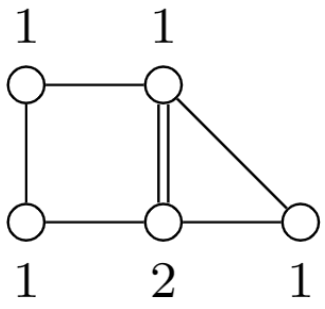
1 1
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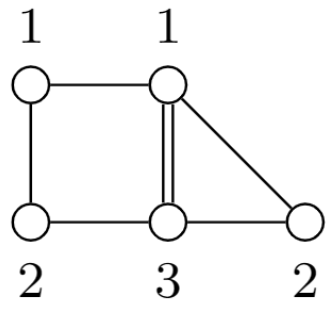
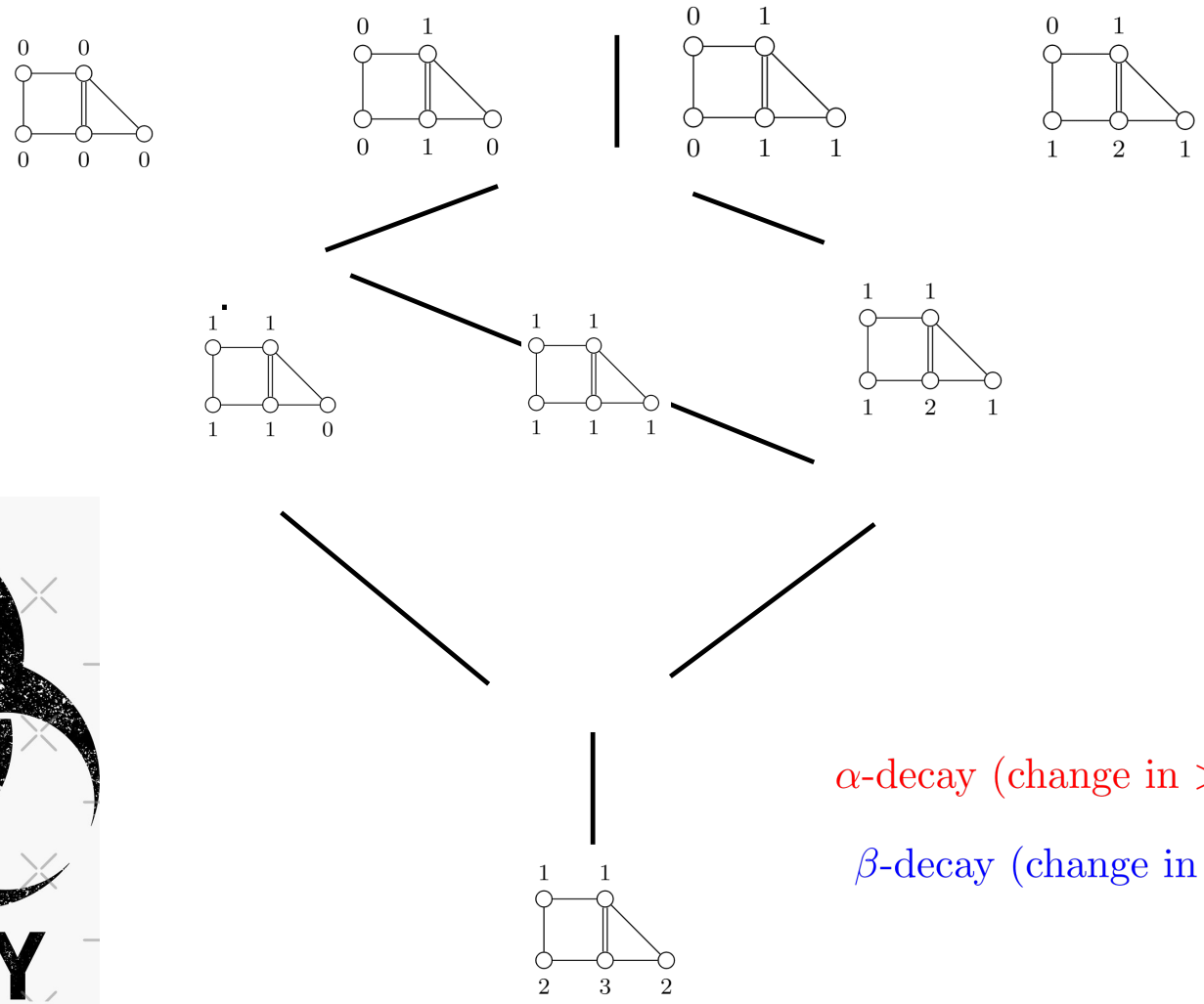
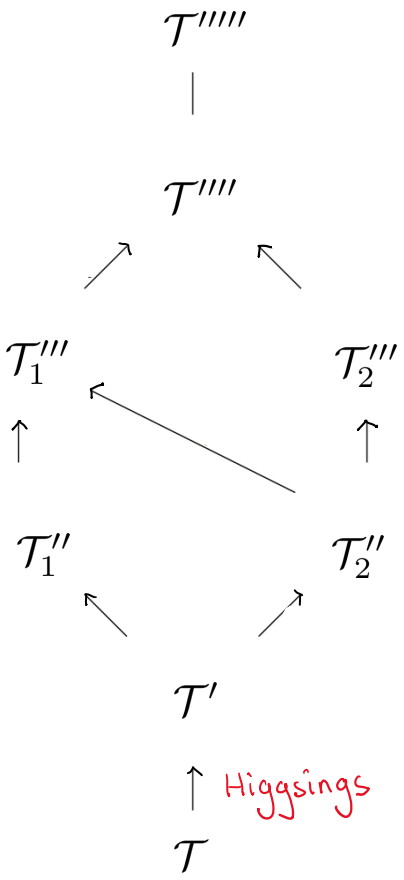

1 1
2 3 2

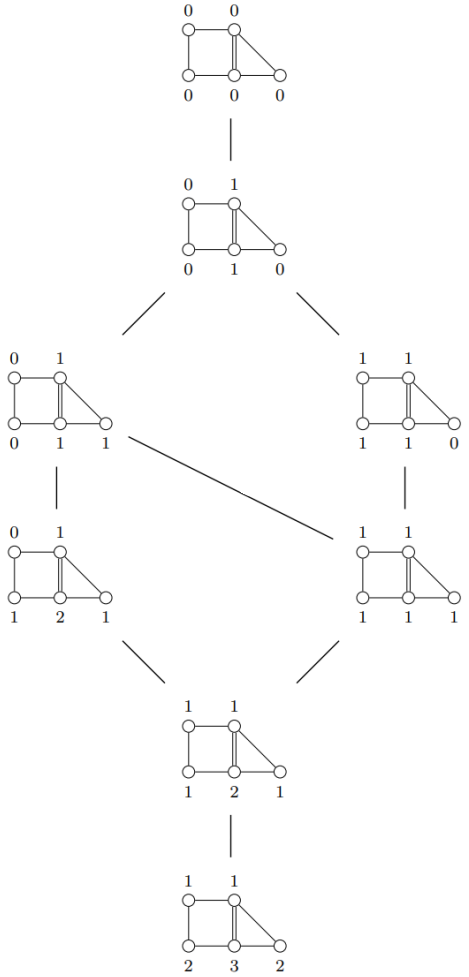


α-decay (change in > 1 dimension)

β-decay (change in 1 dimension)

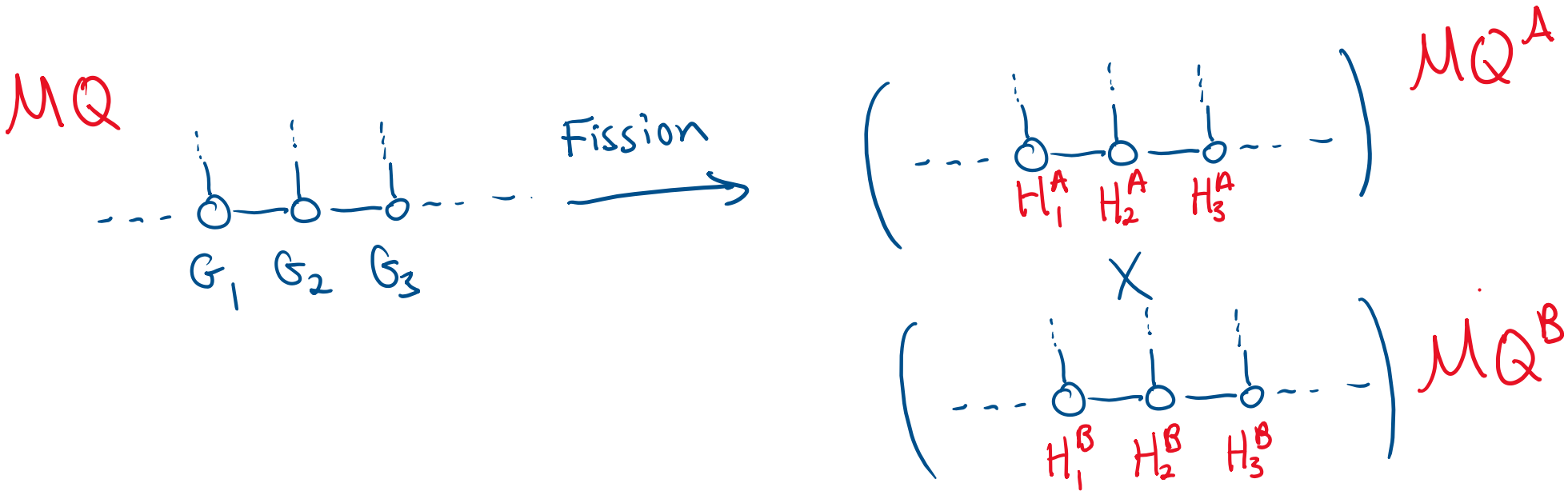


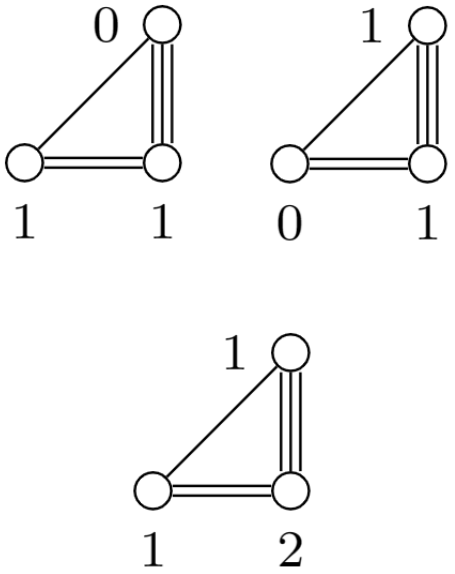
Magnetic Quiver



Fission

Quivers can always be split into a product of two quivers as long as all the gauge groups of the resultings quivers are good and the gauge group ranks are preserved: $G_i = H_i^A + H_i^B$.




0
1
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1
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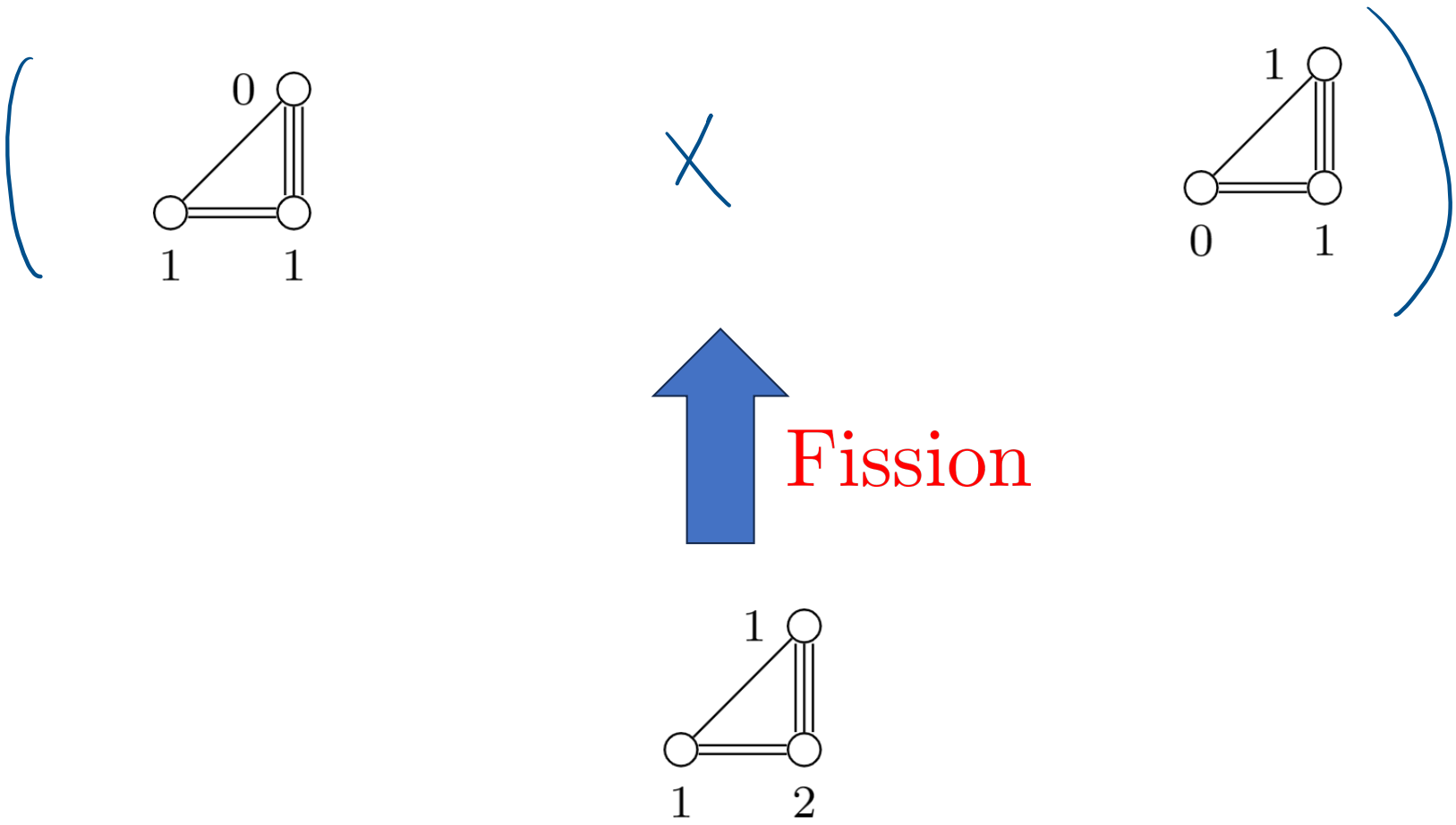
2


FISSION


0
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Fission
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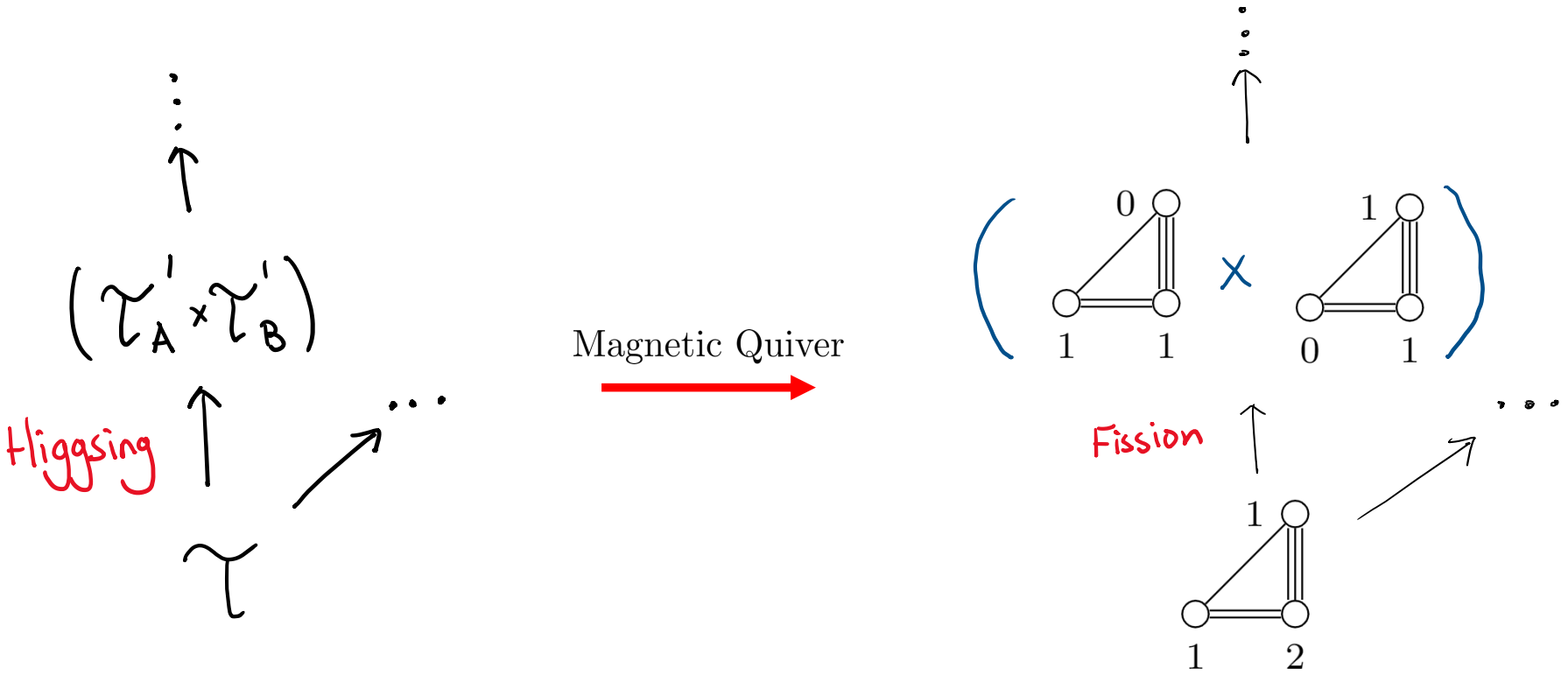
Higgsing
Magnetic Quiver
Fission
0
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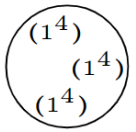
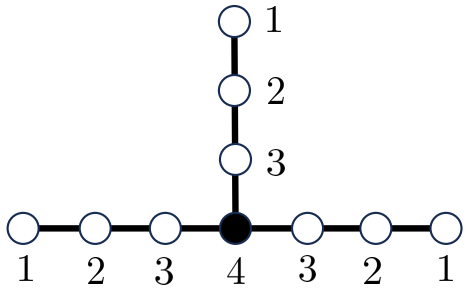
2

Higgsing 4d $\mathcal{N} = 2$ SCFT: T_4



T_4



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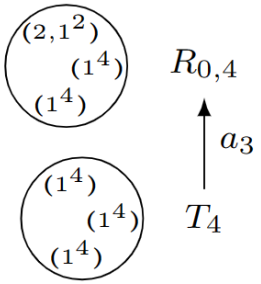

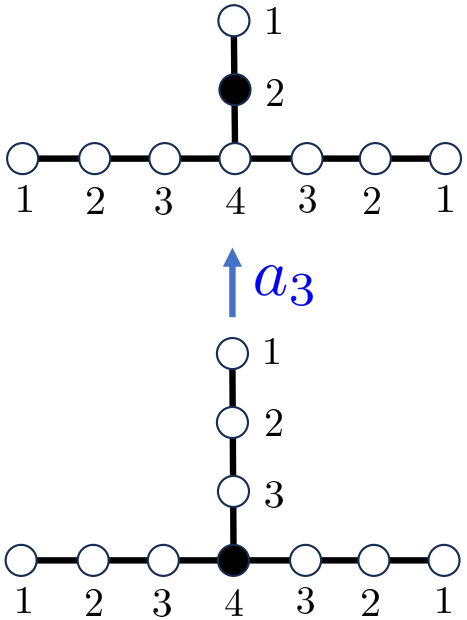

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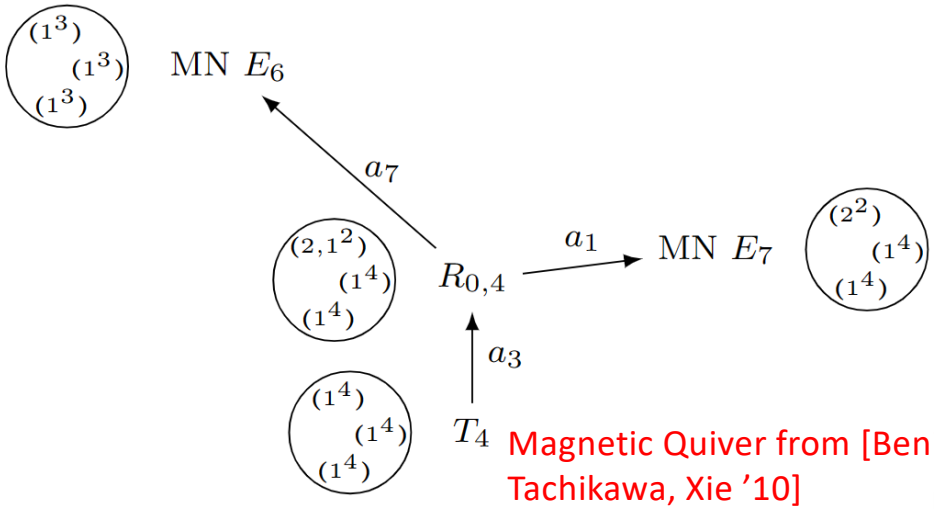
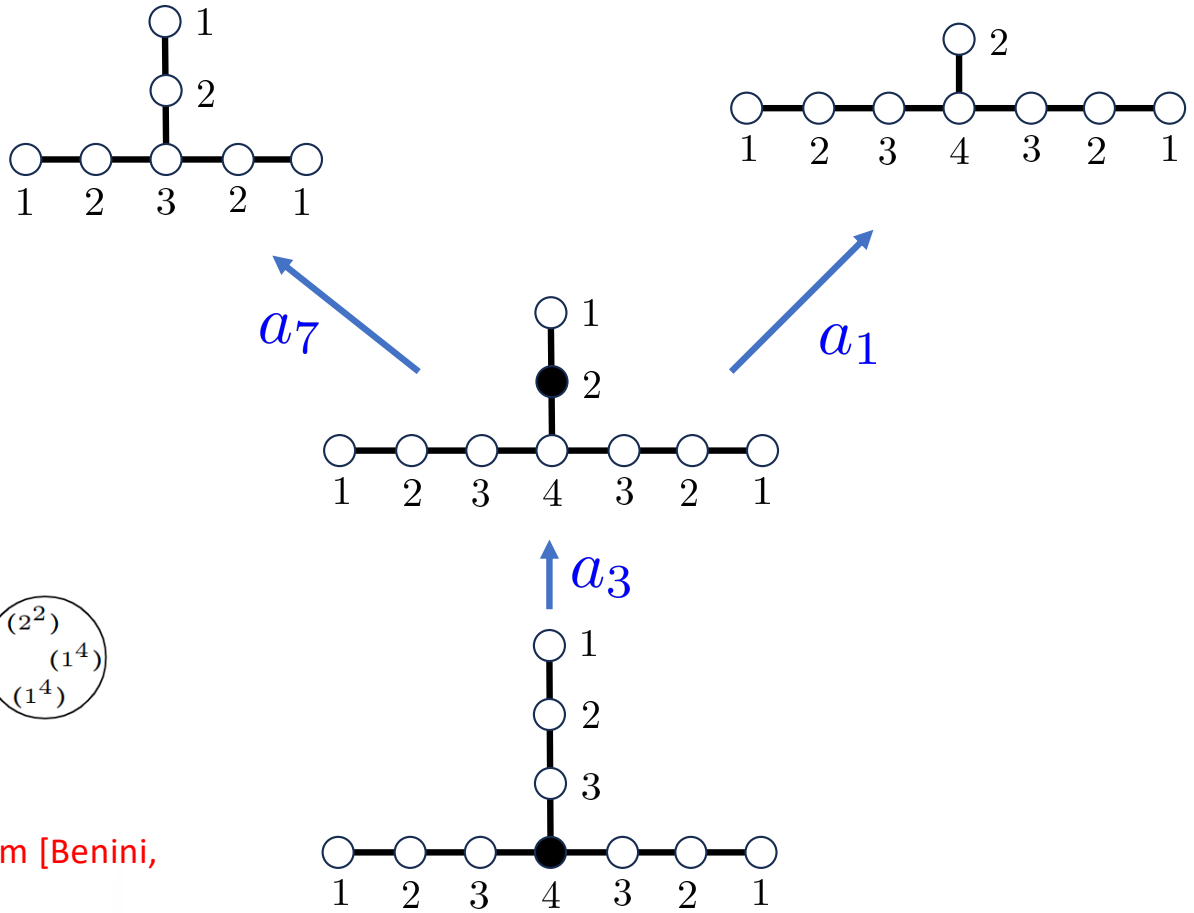


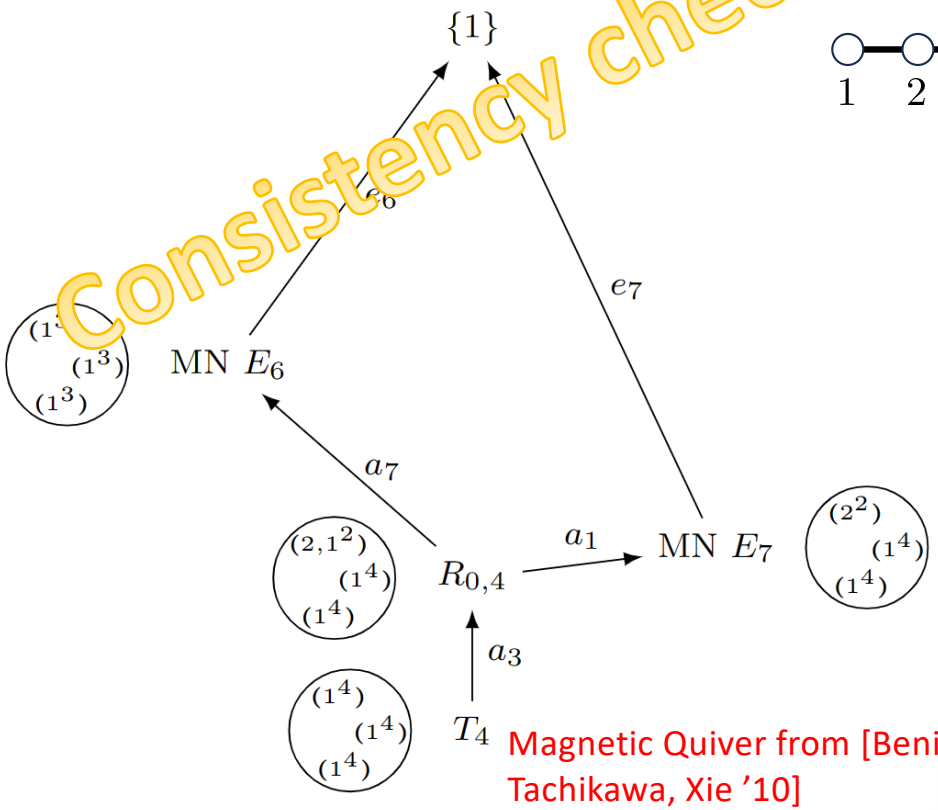
Magnetic Quiver from [Benini, Tachikawa, Xie '10]



Higgsing 4d $\mathcal{N} = 2$ SCFT: T_4

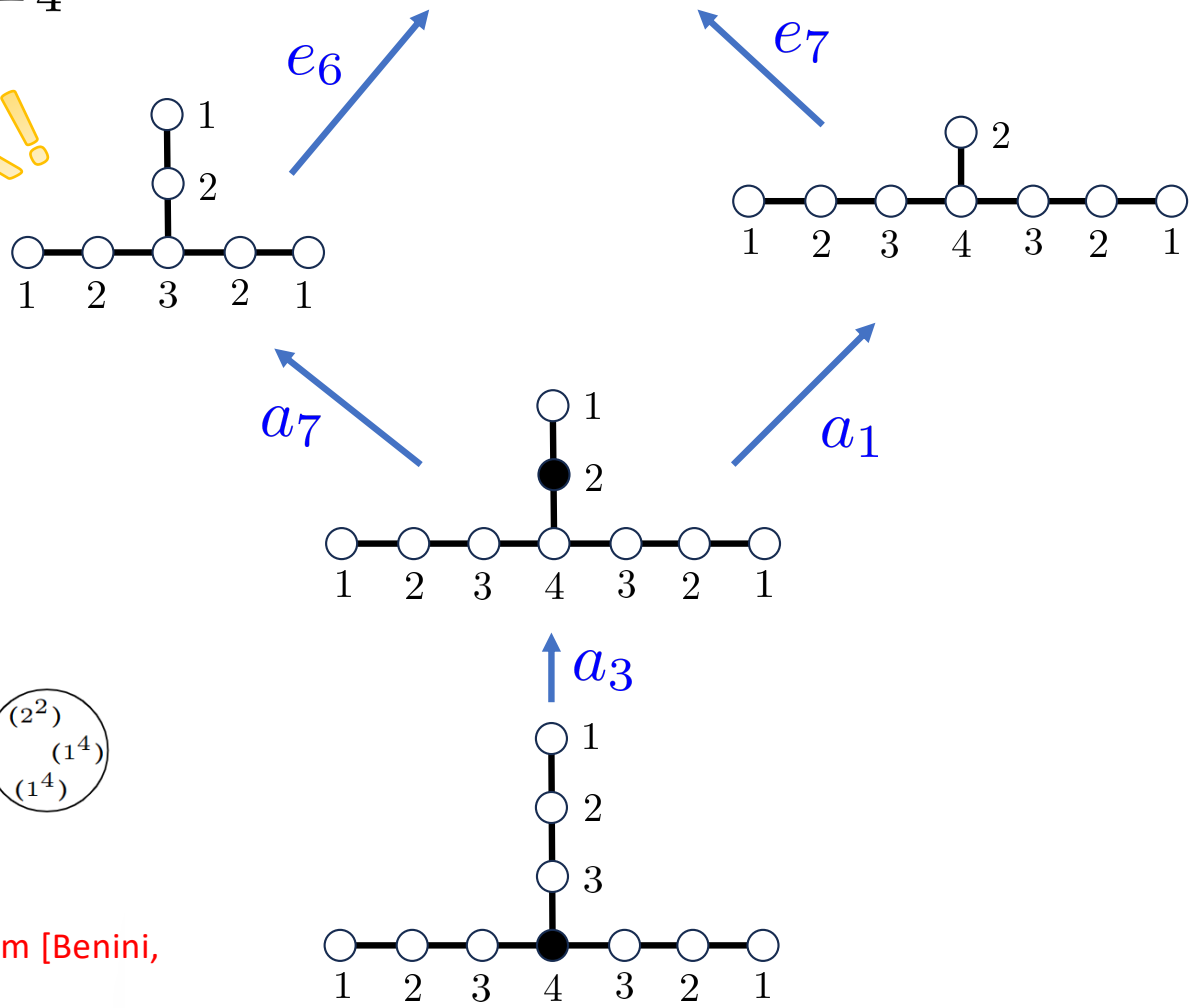
*This example is consistent with closing partial puncture of class $\mathcal{S}$ theories

Consistency check!

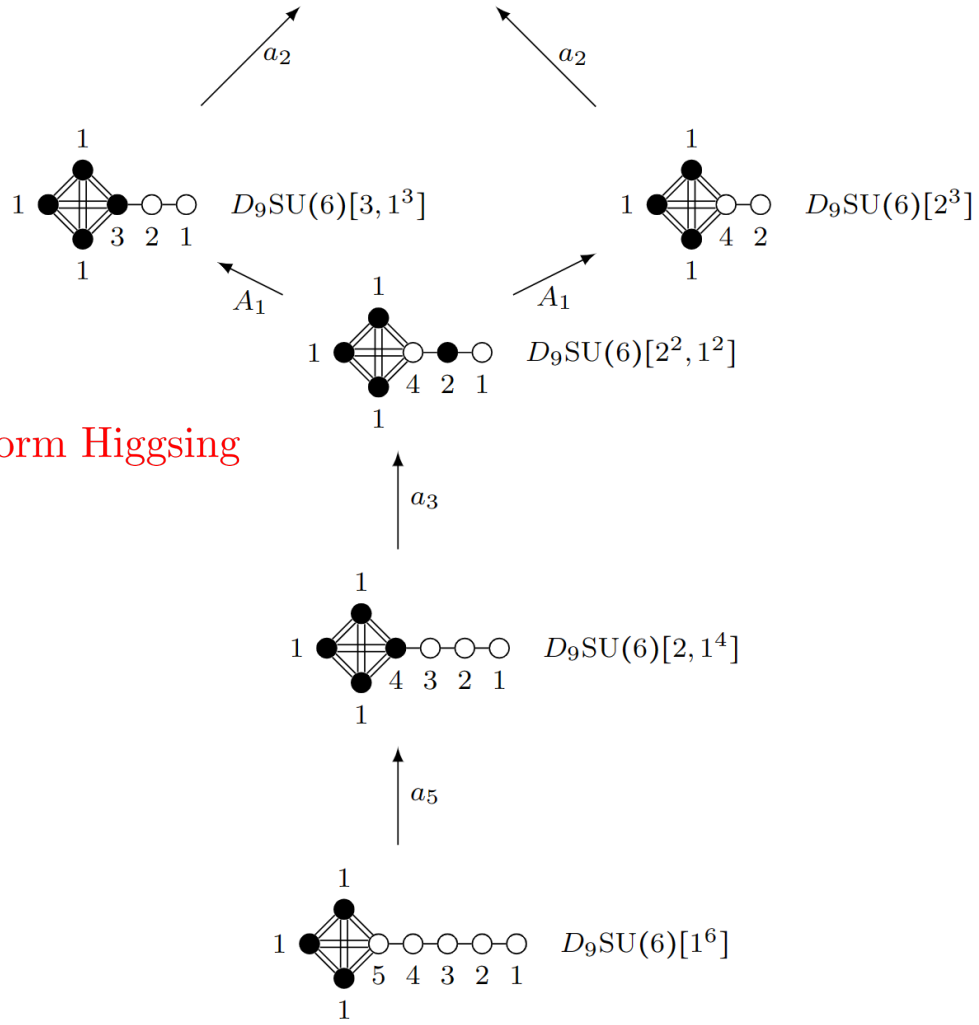


Magnetic Quiver from [Benini, Tachikawa, Xie '10]

Trivial



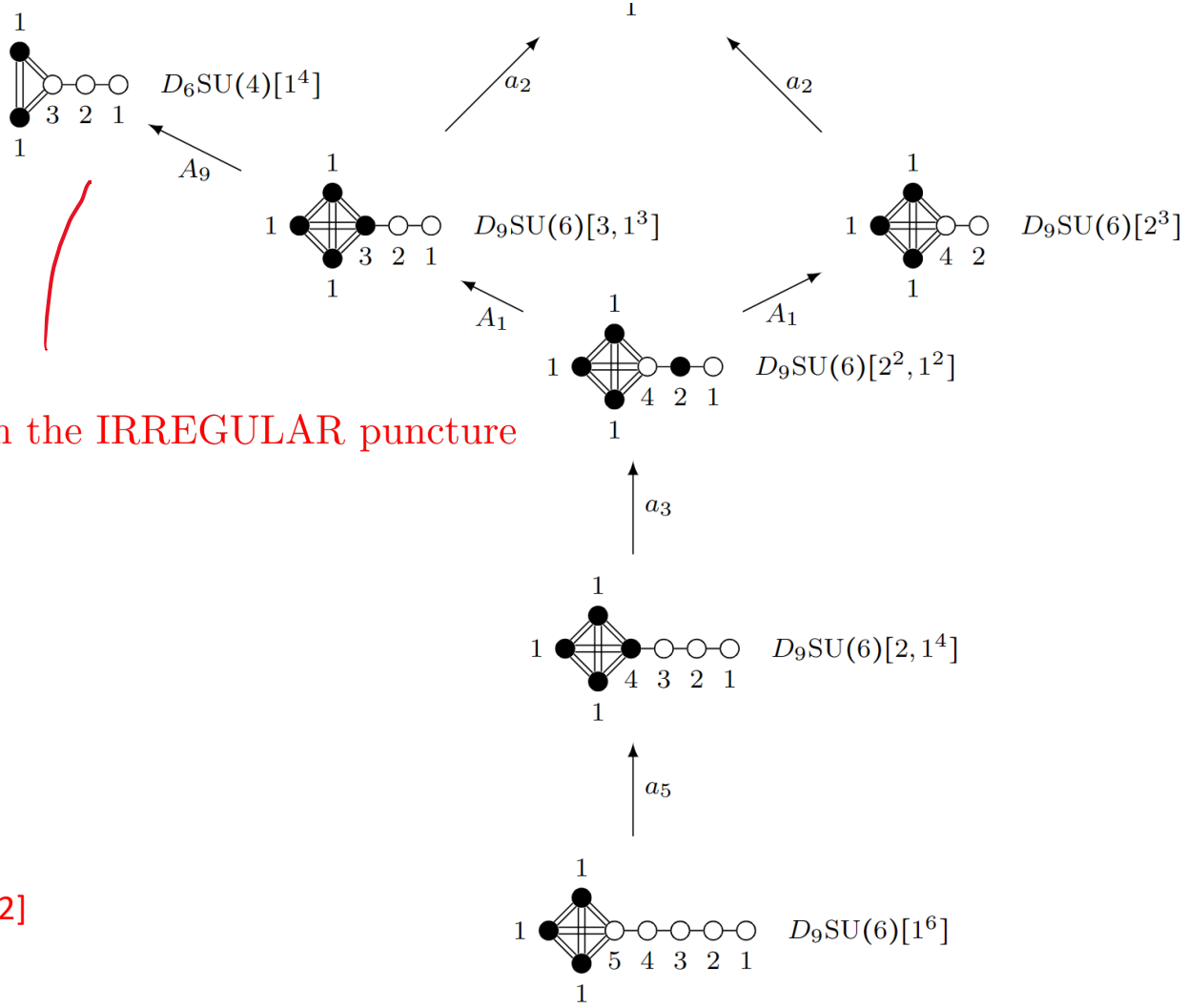
Higgsing 4d $\mathcal{N}=2$ SCFT: Argyres-Douglas Theory: $D_9SU(6)[1^6]$



Closing the regular puncture to perform Higgsing

Magnetic Quiver from [Xie '12]

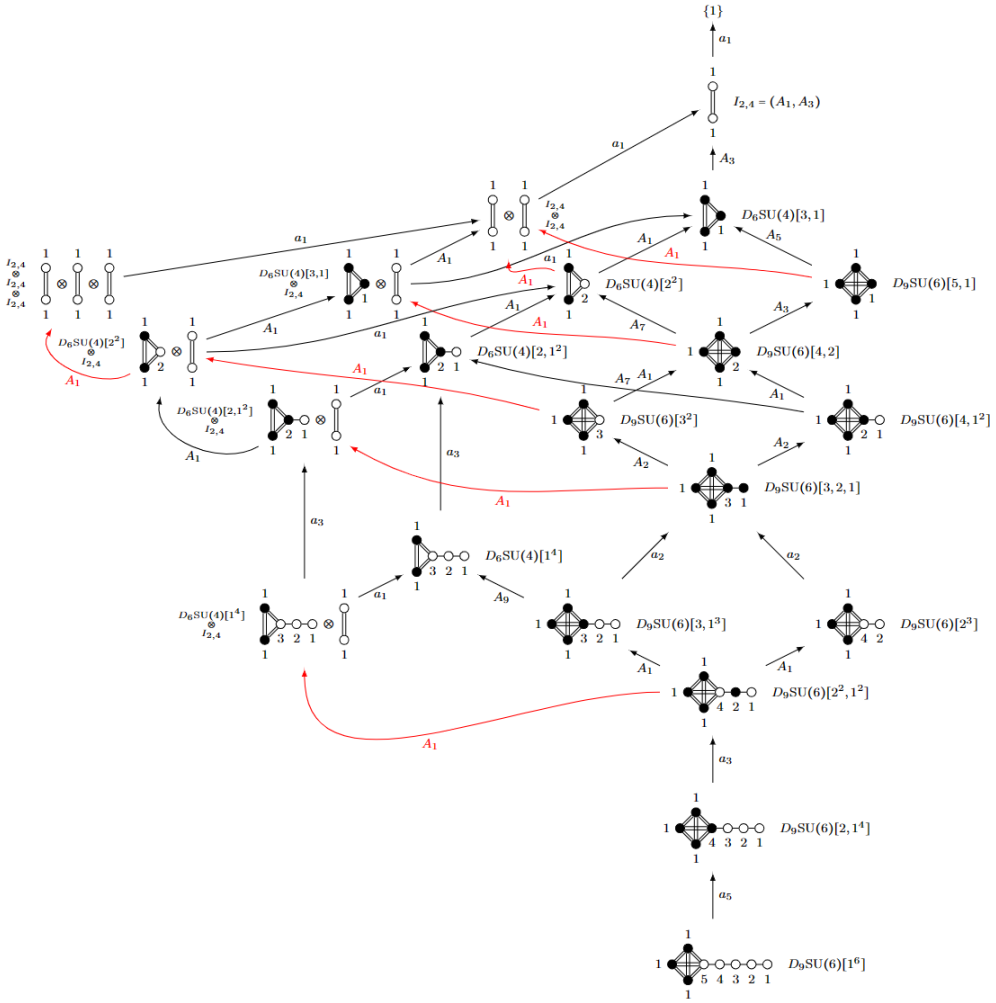
Higgsing 4d $\mathcal{N}=2$ SCFT: Argyres-Douglas Theory: $D_9 SU(6)[1^6]$



This Higgsing is done on the IRREGULAR puncture

Magnetic Quiver from [Xie '12]

Higgsing 4d $\mathcal{N} = 2$ SCFT: Argyres-Douglas Theory: $D_9SU(6)[1^6]$



Magnetic Quiver from [Xie '12]

Rank 2 4d $\mathcal{N} = 2$ SCFT: $Q^2_{A_3}$

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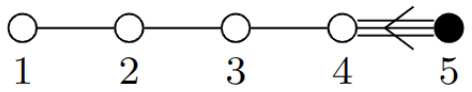
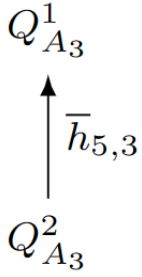
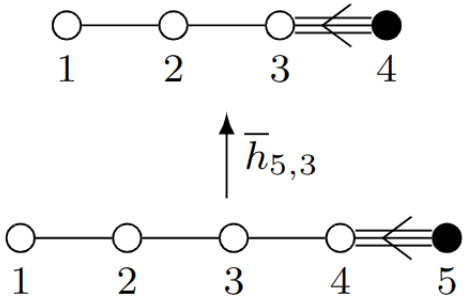


Rank 2 4d $\mathcal{N} = 2$ SCFT: $Q^2_{A_3}$

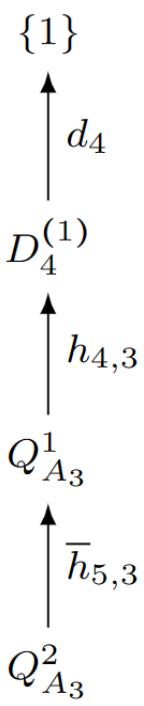
Magnetic Quiver from [Bourget, Grimminger, Hanany, Sperling, Zafrir, Zhong '20]

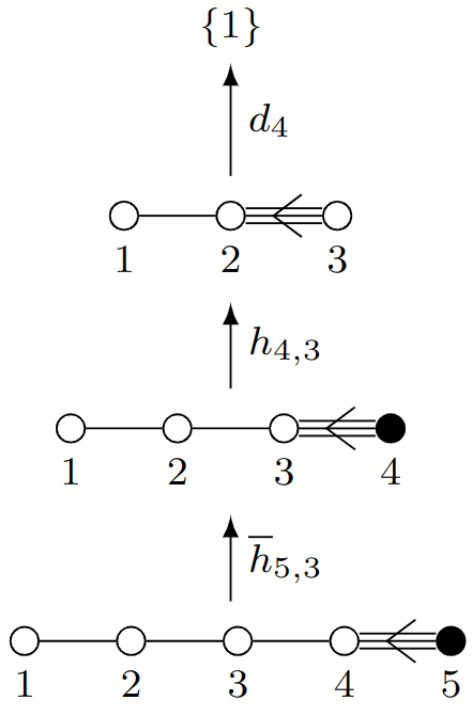




Rank 2 4d $\mathcal{N} = 2$ SCFT: $Q^2_{A_3}$

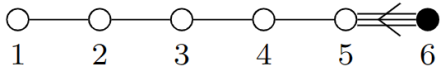
Magnetic Quiver from [Bourget, Grimminger, Hanany, Sperling, Zafrir, Zhong '20]





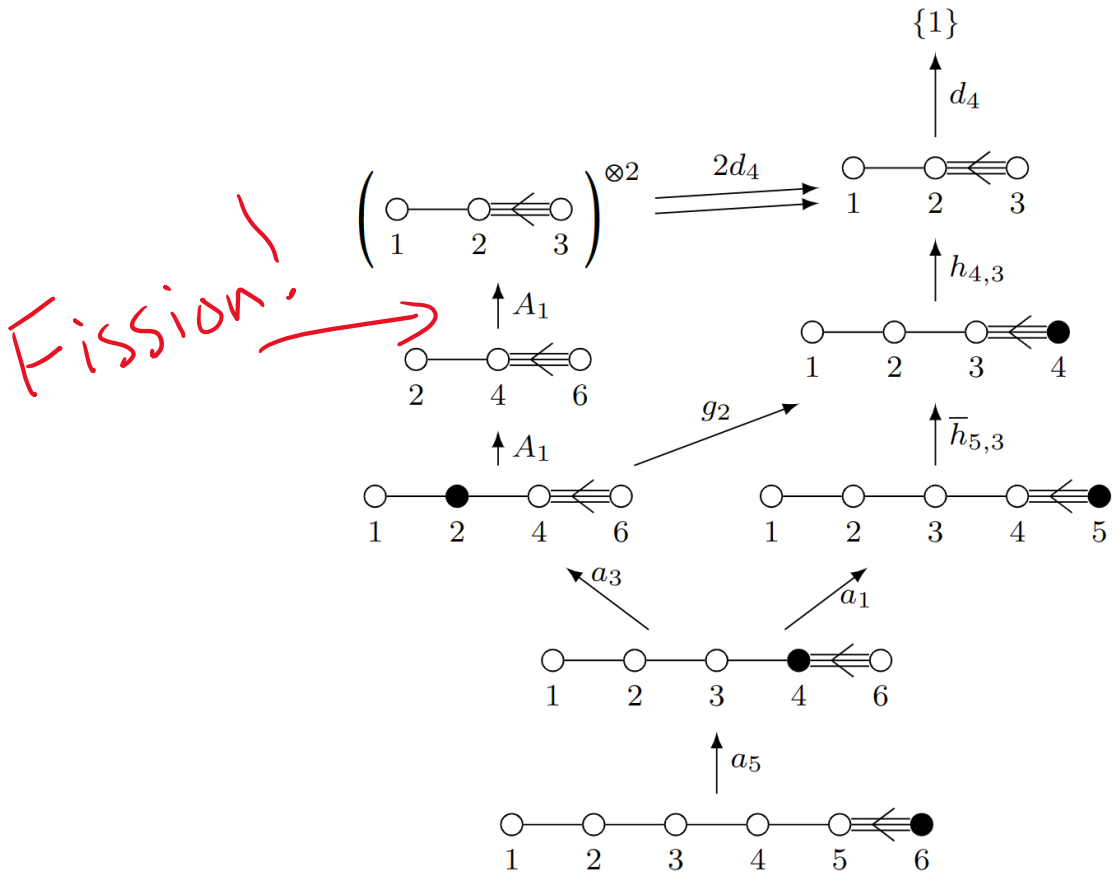
Rank 3 4d $\mathcal{N} = 2$ SCFT: $Q^3_{A_3}$

$Q^3_{A_3}$



Rank 3 4d $\mathcal{N} = 2$ SCFT: $Q^3_{A_3}$

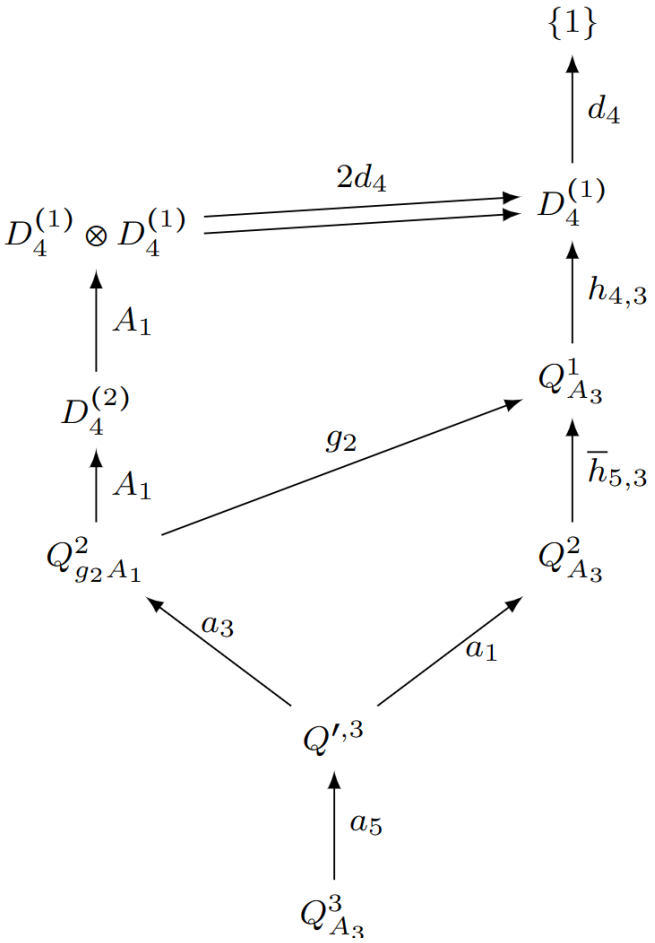
Magnetic Quiver from [Bourget, Grimminger, Hanany, Sperling, Zafrir, Zhong '20]

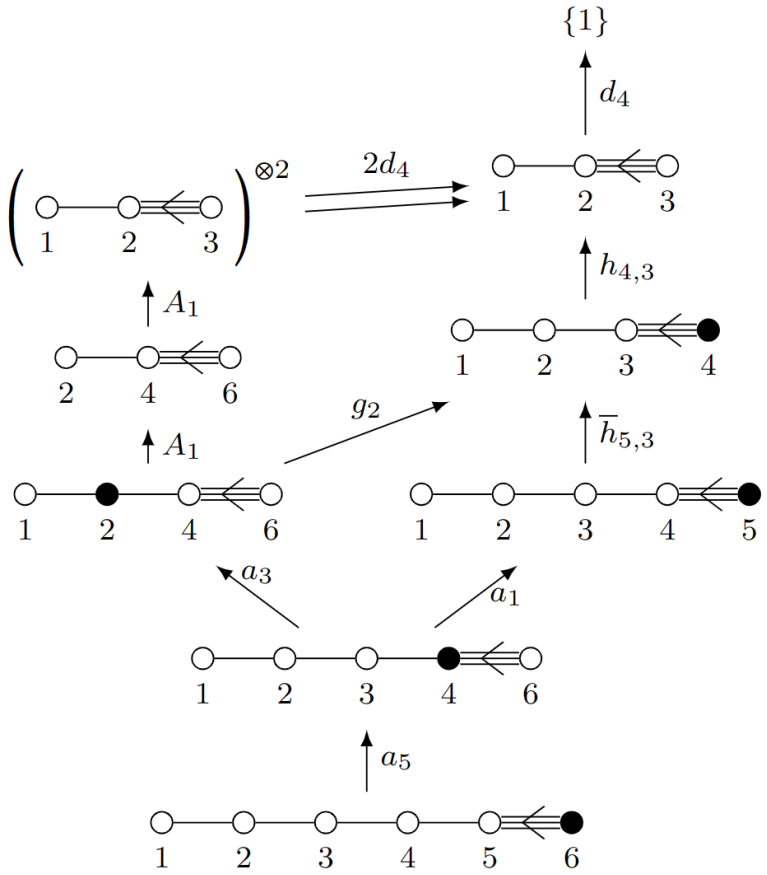


$Q^3_{A_3}$

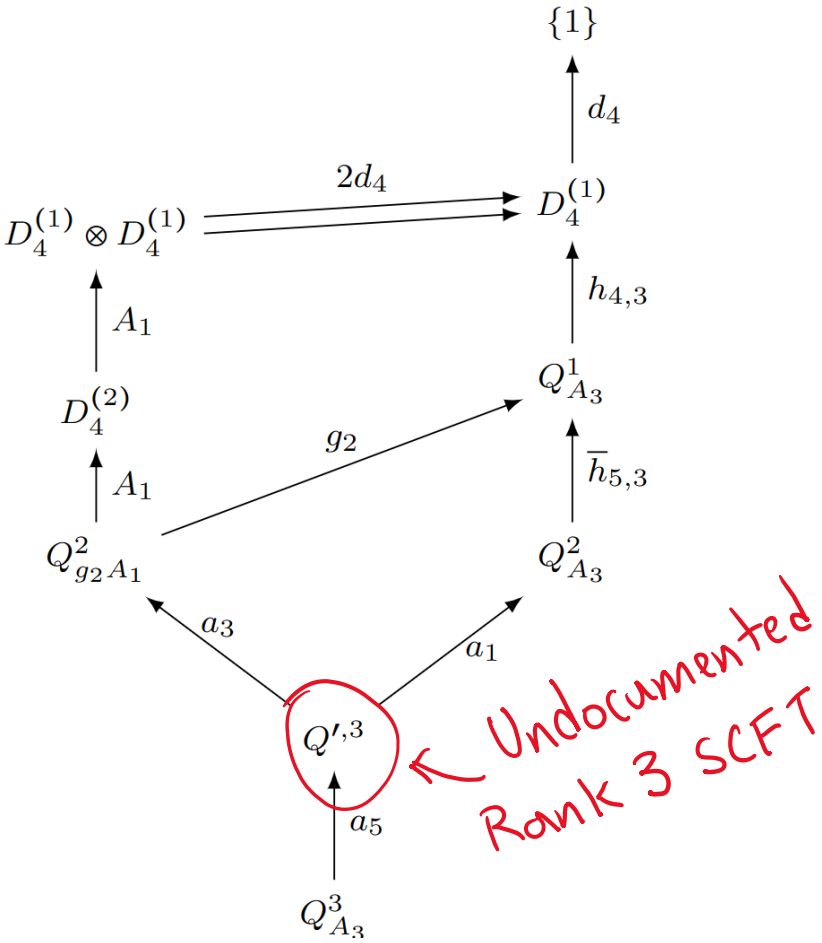
Rank 3 4d $\mathcal{N}=2$ SCFT: $Q^3_{A_3}$

Magnetic Quiver from [Bourget, Grimminger, Hanany, Sperling, Zafrir, Zhong '20]

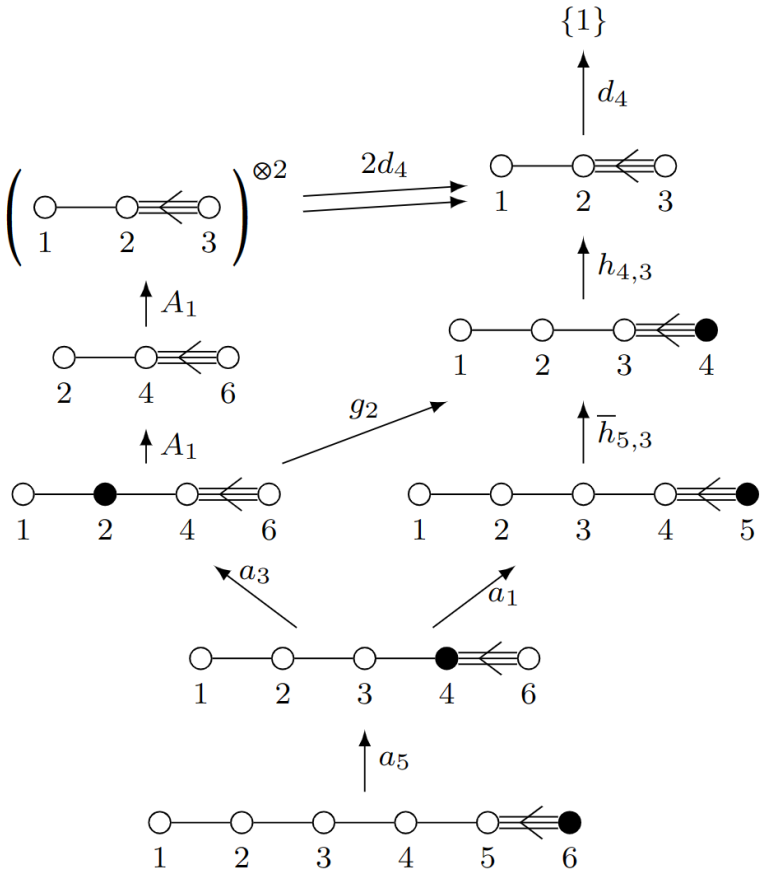




Rank 3 4d $\mathcal{N} = 2$ SCFT: $Q^3_{A_3}$



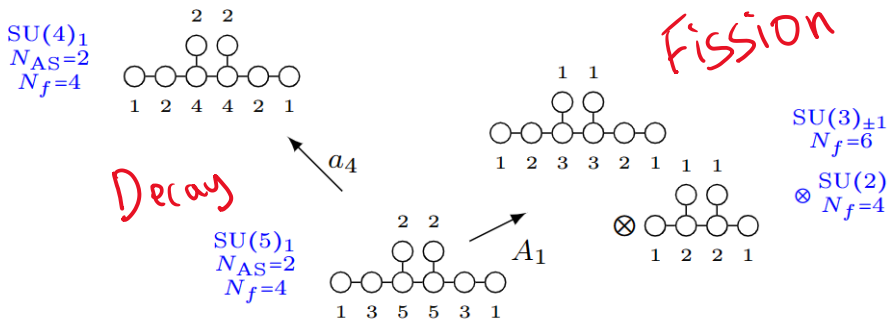
Magnetic Quiver from [Bourget, Grimminger, Hanany, Sperling, Zafrir, Zhong '20]



Higgsing 5d $\mathcal{N}=1$ SCFT: $SU(5)_1$, $N_f=4$, $N_{\rm AS}=2$

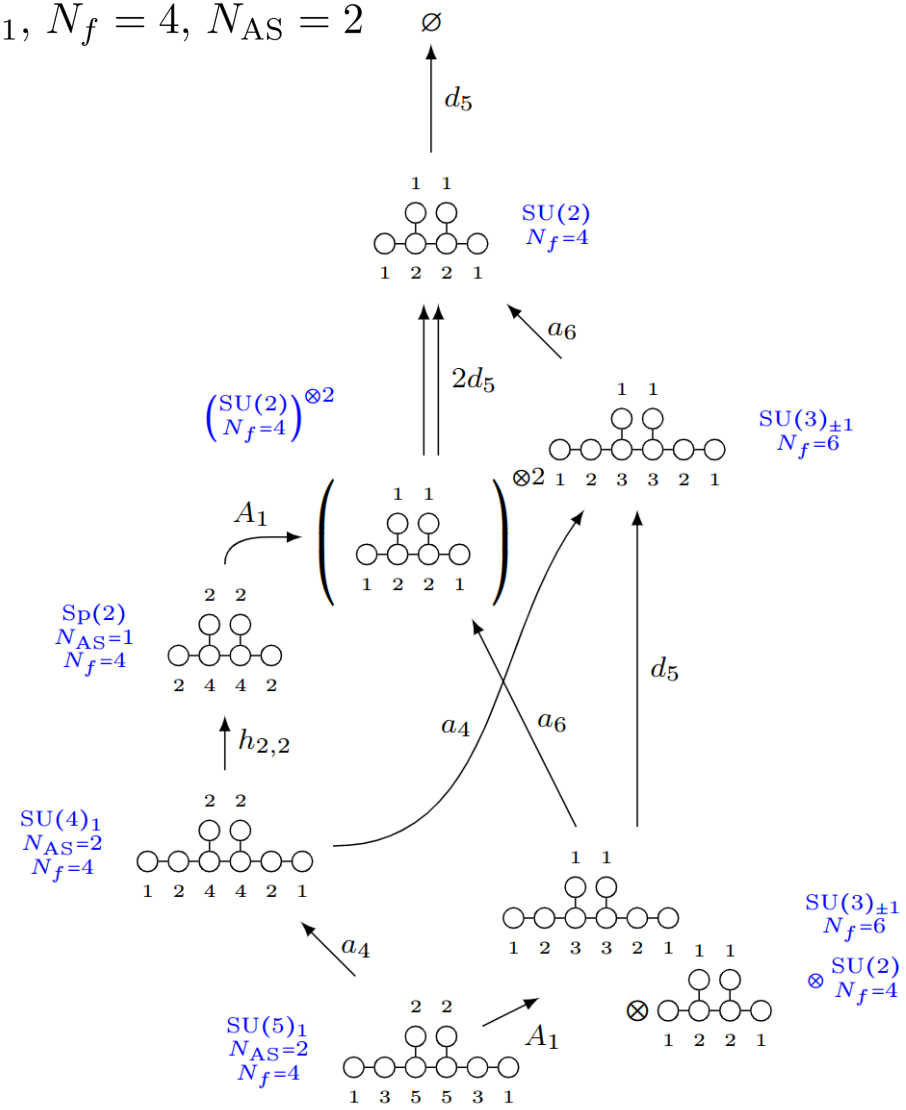
In 5d, Higgsings now involve giving VEVs to massless instantons as well!

[See Sung-Soo and Futoshi's talks]



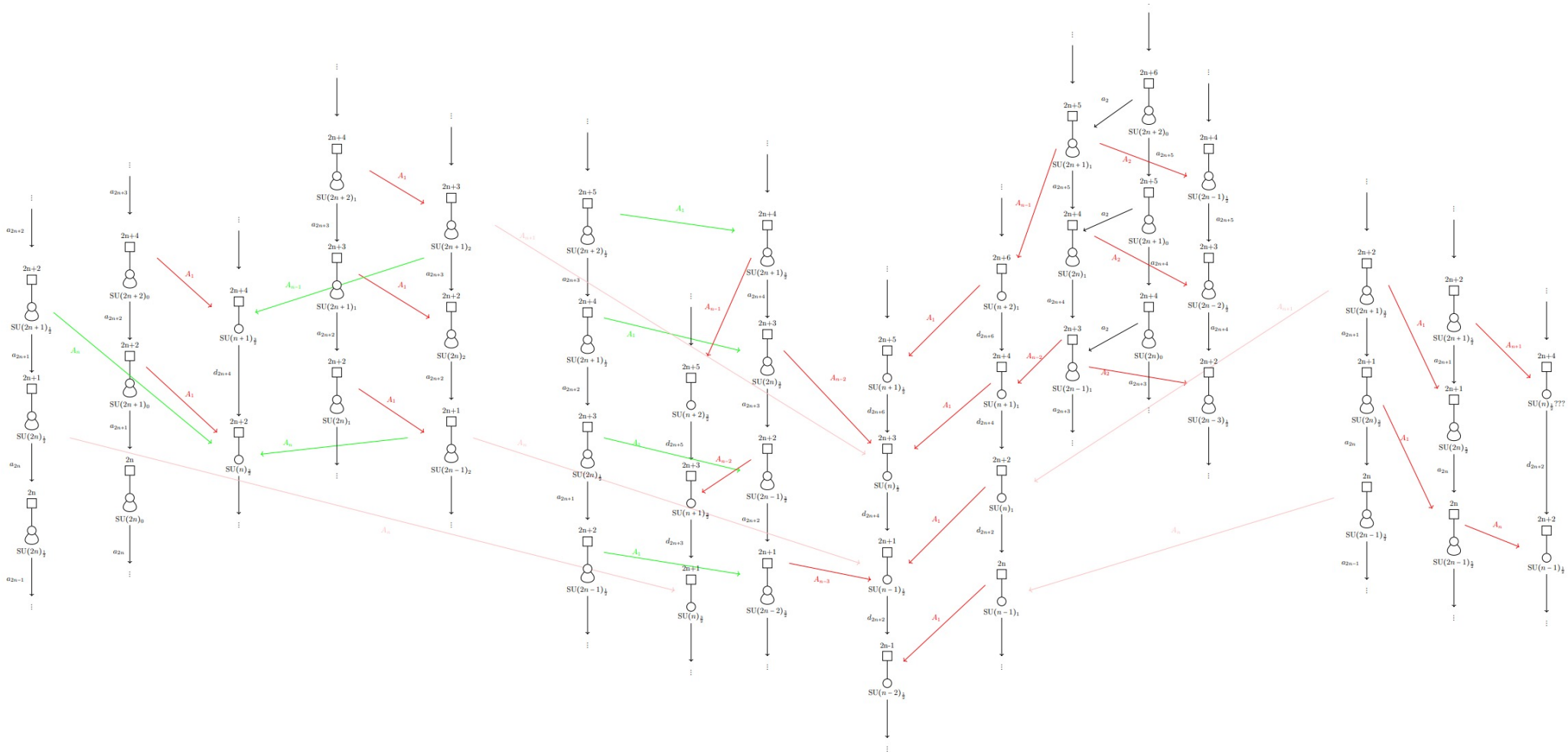
Magnetic Quiver from [Van Beest, Bourget,Eckhard,Schäfer-Nameki '20]

Higgsing 5d $\mathcal{N}=1$ SCFT: $SU(5)_1$, $N_f=4$, $N_{\rm AS}=2$



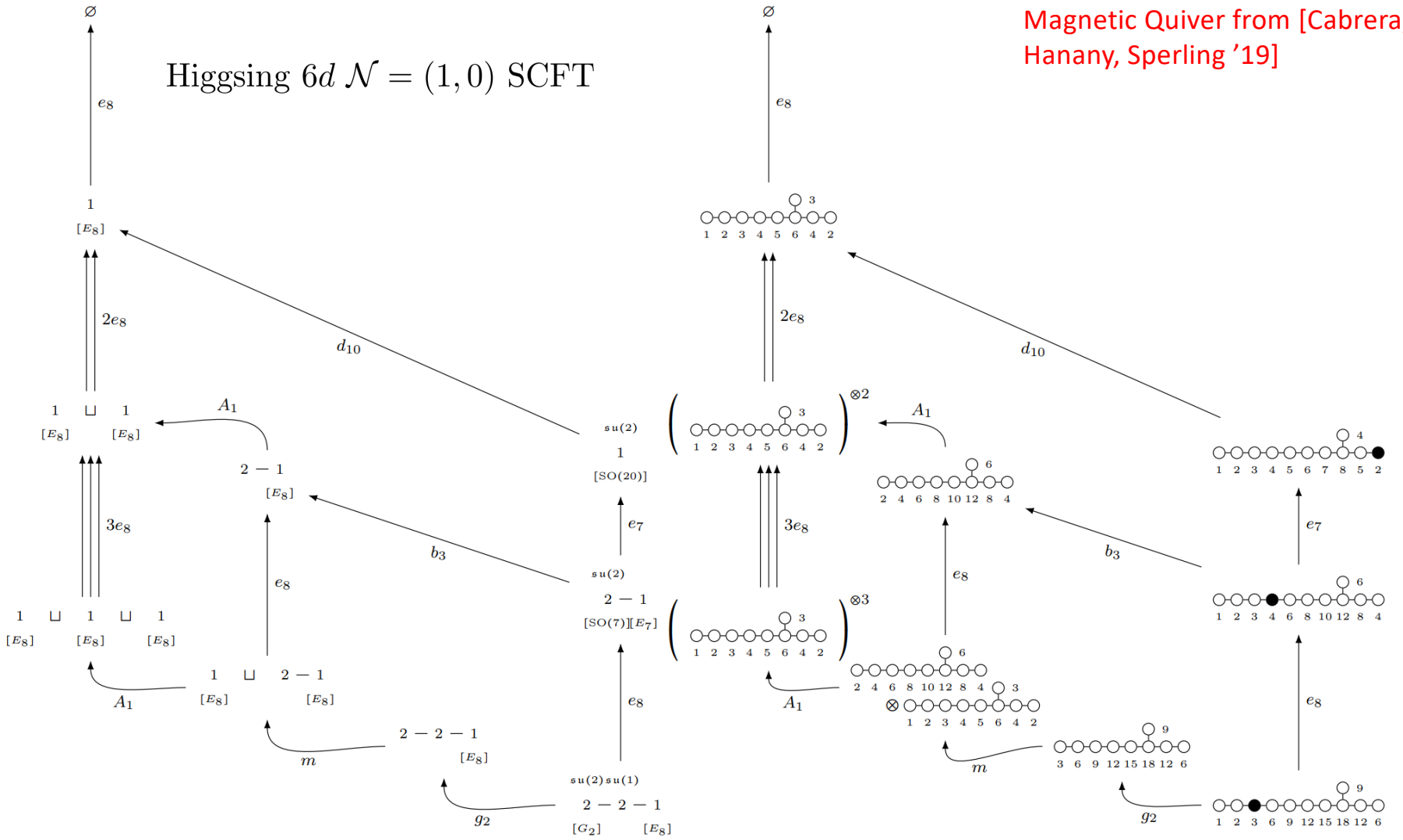
Magnetic Quiver from [Van Beest, Bourget,Eckhard,Schäfer-Nameki '20]

Various 5d $\mathcal{N} = 1$ SCFTs flow to each other



Higgsing $6d$ $\mathcal{N} = (1,0)$ SCFT

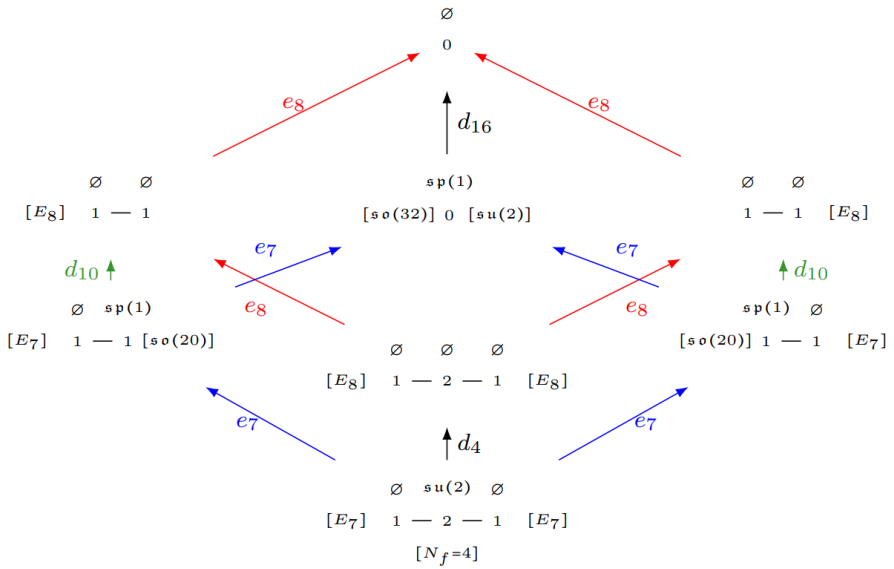
Magnetic Quiver from [Cabrera, Hanany, Sperling '19]

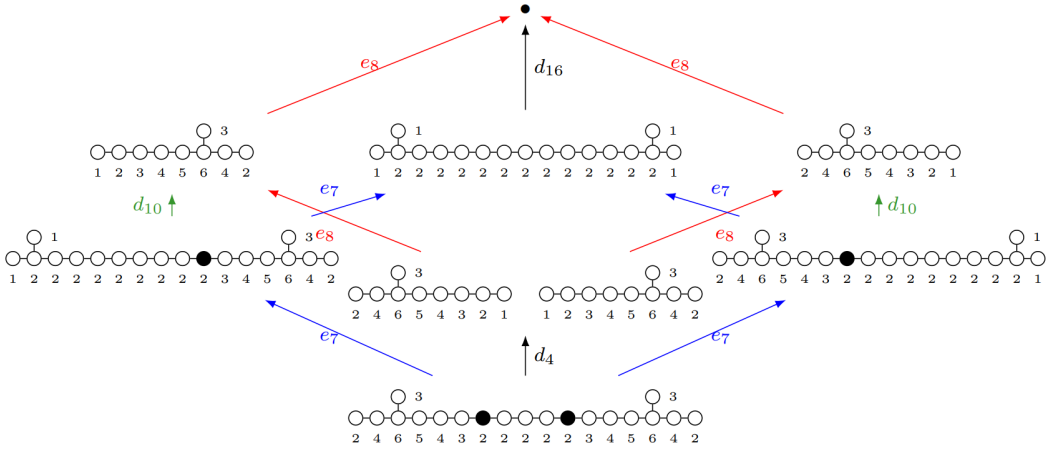


(a) Higgsing

(b) Decay and fission

Higgsing Little String Theories





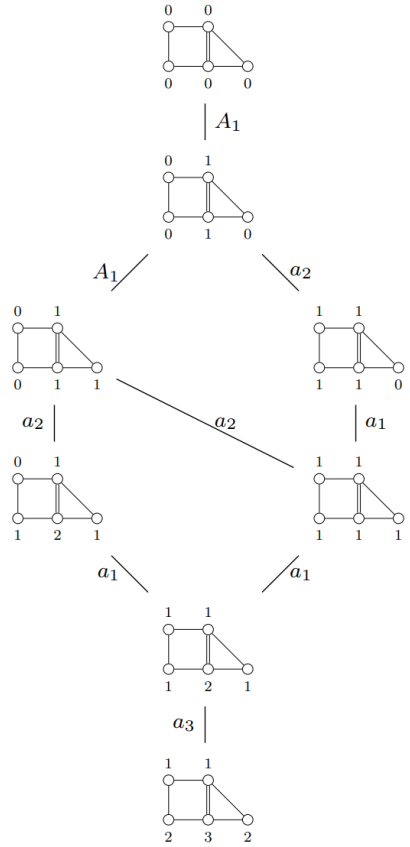
Magnetic Quiver from [Del Zotto, Fazzi, Giri '23]

Advantages of Decay and Fission

- Very easy to carry out (especially with our amazing mathematica code)
- Most methods of Higgsing (group theoretic, closing partial puncture etc) lacks one or more of the following:
  - Requires a Lagrangian description
  - Does not provide minimal Higgsings
  - Does not give the more obscure Higgsings (like the ones via fission)
  - Restricted to particular dimension or families of theories
  - Does not tell you about the transverse space geometry

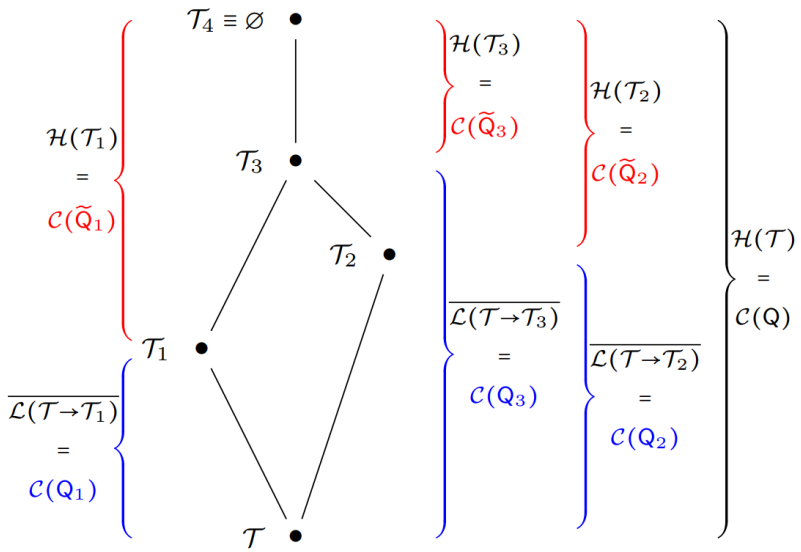
Further Applications

- Elementary transitions



Further Applications

- Elementary transitions
- Identify arbitrary transverse spaces



Decay and Fission Algorithm

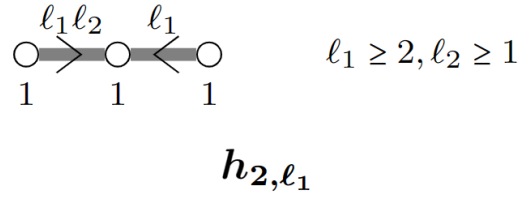
$\widetilde{\mathsf{Q}}_i \leftrightarrow$ magnetic quiver of $\mathcal{T}_i$

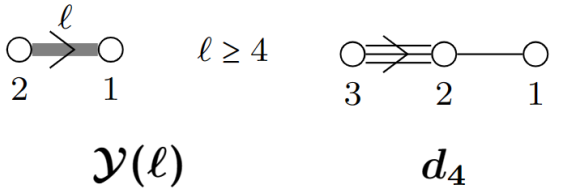
Standard Quiver Subtraction

$\mathsf{Q}_i \leftrightarrow \overline{\text{symplectic leaf } \mathcal{L}(\mathcal{T}\to\mathcal{T}_i)}$

Further Applications

- Elementary transitions
- Identify arbitrary transverse spaces
- Discover new isolated symplectic singularities





Future

- Find flows between seemingly different families of SCFTs
- Identify missing SCFTs
- Bootstrapping new SCFTs through reversed process: UnHiggsing
- Test maximization theorems (a, c, F)