Branes and the Swampland

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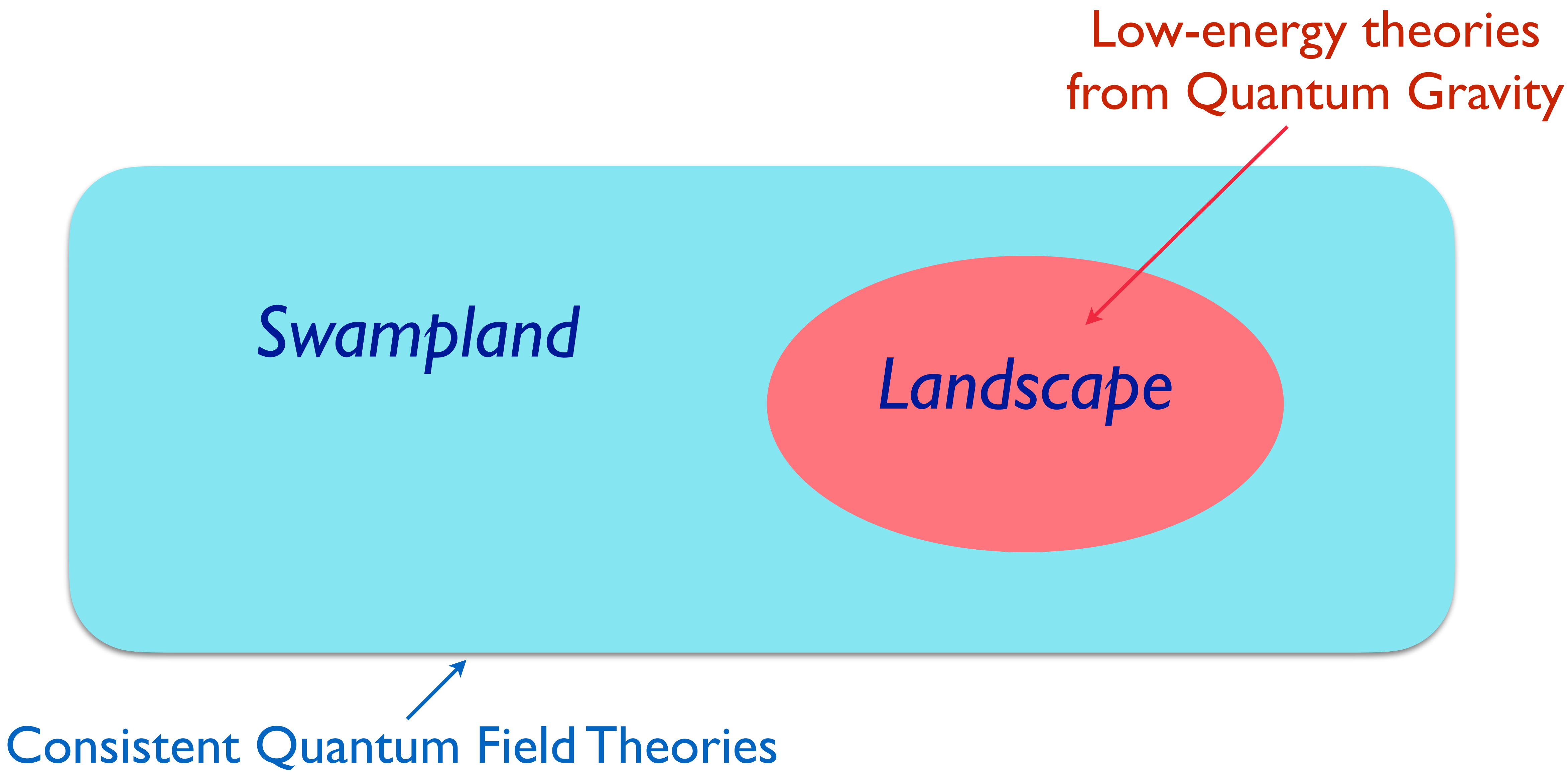
POSTECH

Based on

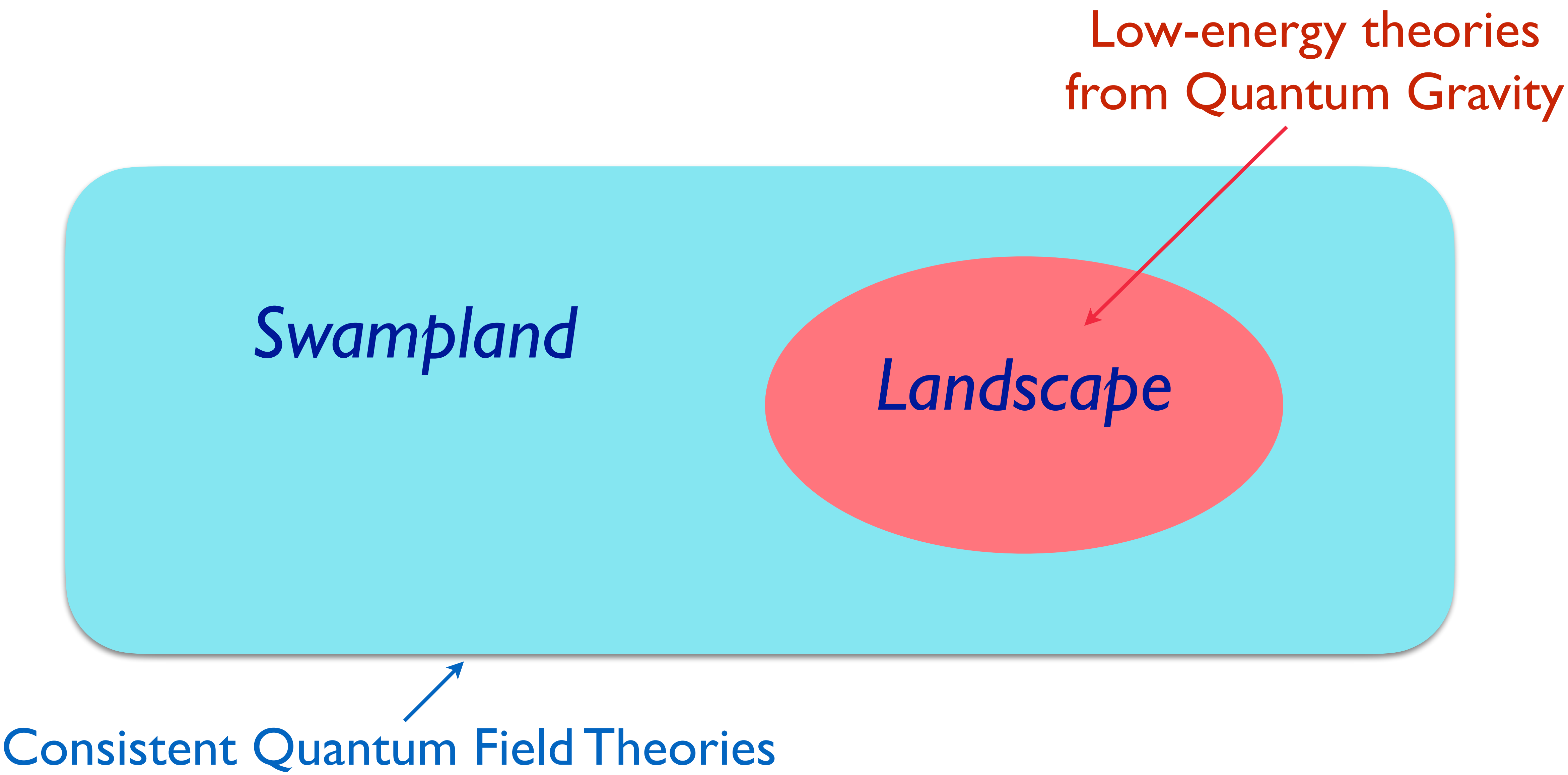
arXiv : 1905.08261 with Gary Shiu and Cumrun Vafa
arXiv : 1912.06144 with Houri Tarazi and Cumrun Vafa
arXiv : 2004.14401 with Sheldon Katz, Houri Tarazi and Cumrun Vafa

The Swampland



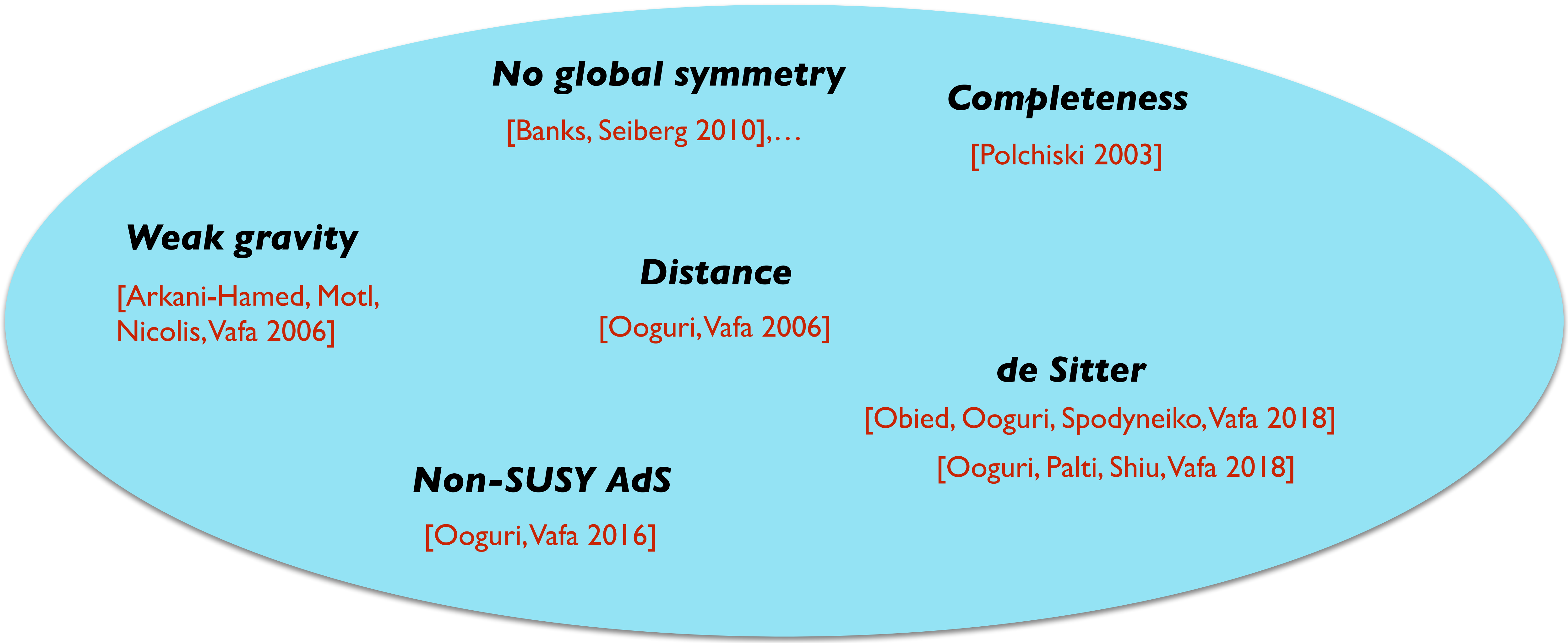
- Landscape : QFTs arising in low energy limit of quantum gravity.
- Swampland : Consistent low-energy QFTs that cannot be completed by quantum gravity in UV.

The Swampland



- Swampland program aims to find conditions that can distinguish QFTs in the landscape from those in the Swampland, based on Black hole physics, unitarity, etc.

Swampland conjectures



- Completeness conjecture : Spectrum of charges for every gauge field in a gravitational theory is complete.

- Distance conjecture : At an infinite distance in moduli space, a tower of infinitely many light states should emerge.

String Lamppost Principle (SLP)

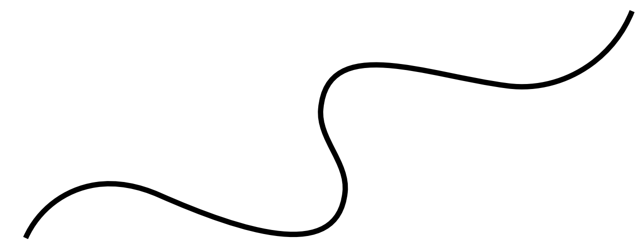
- String Lamppost Principle : All consistent quantum theories of gravity can be captured in string theory.

- For example,
    - N=32 low-energy QFTs are completely fixed by supersymmetry.
        - 11d supergravity is UV-completed by M-theory
        - 10d IIA/IIB supergravities are realized by IIA/IIB String theories
    - 6d N=(2,0) supergravity is also uniquely fixed by supersymmetry and anomaly. It arises as low energy QFT of IIB string on K3.

- May be only for large SUSY or chiral QFTs?

Goal for today

- Find more Swampland conditions by using string probes.



Unitarity of worldsheet CFT

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Completeness conjecture → New Swampland conditions

+

Distance conjecture

- We shall focus on SUSY QFTs in Minkowski spacetime with d>3.

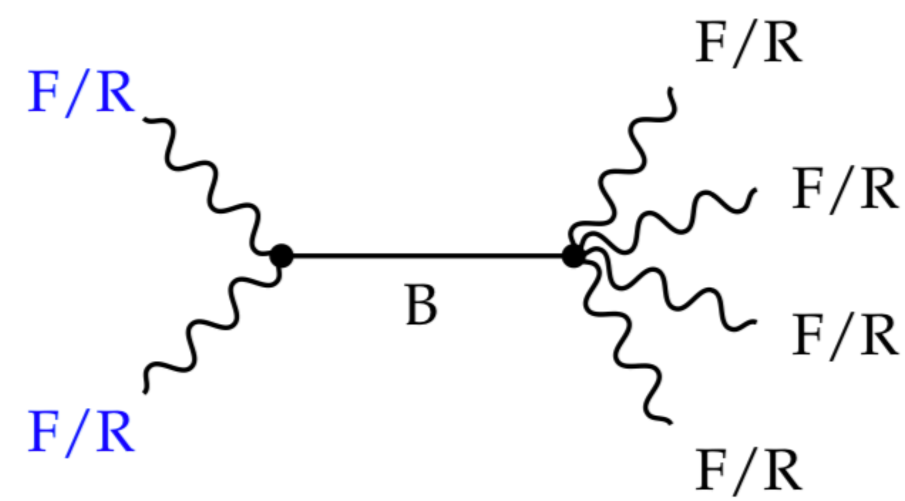
Plan

- 10d Supergravities with 16 SUSY
- 10>d>3 Supergravities with 16 SUSY
- 6d (1,0) Supergravities
- Summary and Outlook

10d N=(1,0) Supergravity

10d N=(1,0) Supergravity

- Anomaly cancellation by Green-Schwarz mechanism allows only 4-choices of gauge groups. [Green, Schwarz 1984]



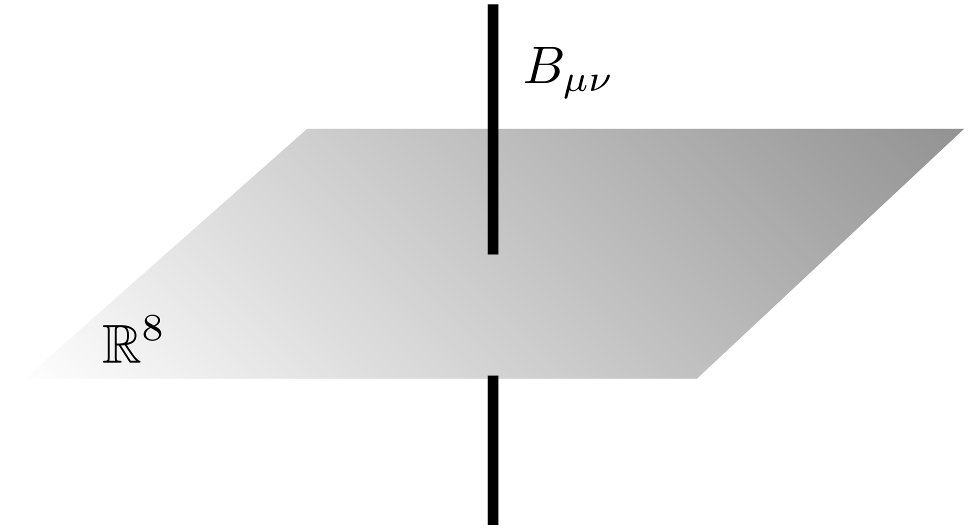
$$SO(32),\ E_8 \times E_8,$$
$$E_8 \times U(1)^{248},\ U(1)^{496}$$

- $SO(32),\ E_8 \times E_8$ models are low energy limits of Heterotic string theories.

- QFTs with gauge group $E_8 \times U(1)^{248}, U(1)^{496}$ cannot be realized in String theory.
  Does this show that SLP is broken? Or these QFTs are inconsistent?

- Check this with string probes.

Strings in 10d (1,0) Supergravity



- Consider 1+1d strings, which are sources for 2-form field $B_{\mu\nu}$ preserving (0,8) SUSY.

- String with tensor charge Q can be introduced by adding to the action

$$S^{\mathrm{str}} = Q \int_{\mathcal{M}_{10}} B_2 \wedge \prod_{a=1}^{8} \delta(x^a) dx^a = Q \int_{\mathcal{M}_2} B_2$$

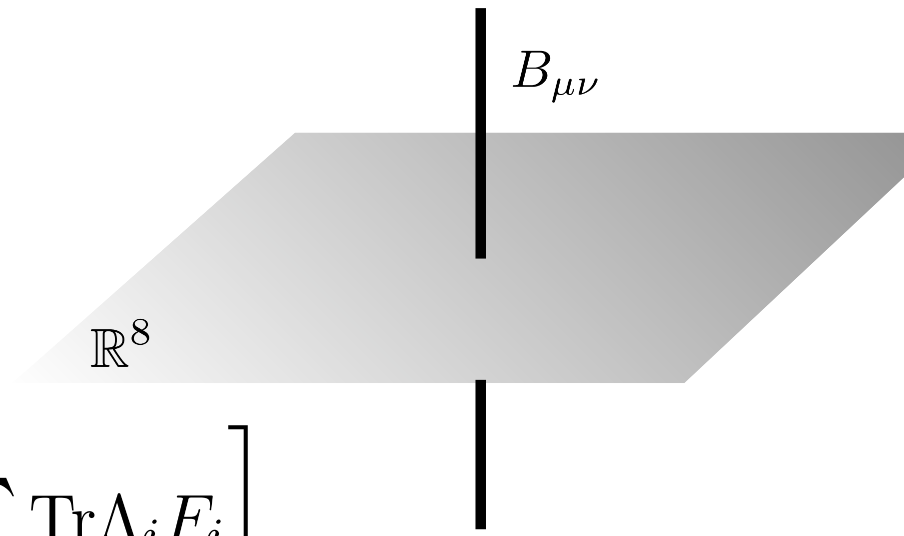
- Completeness of string spectrum charged under the 2-form gauge field ensures that such string states must exist.

Anomaly inflow

- B_2 transforms nontrivially under gauge and local Lorentz symmetry

$$\delta_{\Lambda_i,\Theta} B_2 = a\,\mathrm{tr}\Theta R - \frac{1}{4}\sum_i b_i \mathrm{Tr}\Lambda_i F_i$$

  - Gravitational anomaly cancellation fixes $a = 1$.
  - SUSY invariance fixes $b_i = 1$. [Bergshoeff, de Wit, Nieuwenhuizen 1982]

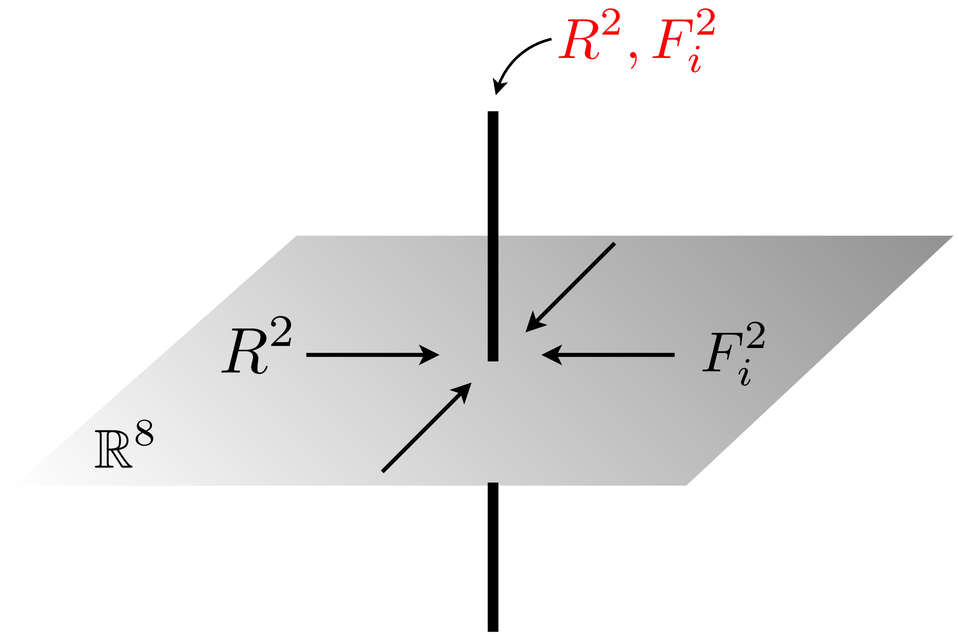


- Anomaly in the presence of a single string.

$$S^{\mathrm{str}} = \int_{\mathcal{M}_2} B_2 \quad \to \quad \delta_{\Lambda,\Theta} S^{\mathrm{str}} = \int_{\mathcal{M}_2} \left[\mathrm{tr}\Theta R - \frac{1}{4}\sum_i \mathrm{Tr}\Lambda_i F_i \right]$$

- This anomaly inflow from bulk gravity theory toward the string worldsheet must be cancelled by anomalies of the worldsheet d.o.f.

Central charges of 2d worldsheet CFT



- In order to cancel anomaly inflows, 2d CFT on a string must have anomalies captured by the anomaly polynomial:

$$I_4 = \frac{1}{2}p_1(T_2) - c_2(SO(8)) + \frac{1}{4}\sum_i \mathrm{Tr}F_i^2$$
$$= \frac{c_L - c_R}{24}p_1(T_2) - \frac{c_R}{12}c_2(SO(8)) + \frac{1}{4}\sum_i k_i \mathrm{Tr}F_i^2$$

- This involves the center of mass modes contribution

C.o.m : $X_{\mu=1,\cdots,8}\ ,\ \lambda_+^{I=1,\cdots,8} \quad \longrightarrow \quad c_L^{\mathrm{com}} = 8\,,\ c_R^{\mathrm{com}} = 12$

- After removing c.o.m part, the interacting CFT has the central charges

$$\hat{c}_L = 16\ ,\ \hat{c}_R = 0\ ,\ k_i = 1$$

't Hooft anomaly for gauge group G_i

Unitary bounds from 2d string probe

- 't Hooft anomaly $k_i = 1 > 0$ implies that the 2d CFT contains the current algebra of the gauge group G_i at level '1' in left-moving sector.

- Unitary representations of level k current algebra of gauge group G has non-trivial central charge contribution.

$$c_G = \frac{k \cdot \mathrm{dim}G}{k + h^\vee} = \begin{cases} 8 & \text{for } E_8 \\ 16 & \text{for } SO(32) \\ 1 & \text{for } U(1) \end{cases}$$

(note, $r_G \leq c_G$)

- Thus we find the unitary condition for a worldsheet CFT such that it can host unitary representations of the current algebras of G_i at level $k_i = 1$:

$$\sum_i c_{G_i} = \sum_i r_{G_i} \leq \hat{c}_L$$

Swampland conditions from strings

- Condition for 2d worldsheet CFT being unitary is

$$\sum_i c_{G_i} = \sum_i r_{G_i} \leq \hat{c}_L$$

16

- Therefore, the string state can consistently couple to the 10d supergravity theory only if $r_G \leq 16$.

- The unitarity of 2d worldsheet CFT together with completeness conjecture rules out 10d supergravity theories with $r_G > 16$.

- The 10d theories with $E_8 \times E_8, SO(32)$ saturate the rank bound.

- On the other hand, the 10d theories with $E_8 \times U(1)^{248}, U(1)^{496}$ violates the rank bound, and therefore they belong to the Swampland.

(another argument is given in [Adams, DeWolfe, Taylor 10])

SLP holds in 10d (1,0) SUGRA!

Supergravities with 16 SUSY in d<10

Supergravity theories in d<10 with 16 SUSY

- Supergravity theories with 16 SUSY in dimension d<10 are non-chiral theories (except 6d N=(2,0)). There is no constraint from local anomalies. Therefore the theory can have arbitrary gauge group (up to global anomalies). In particular, arbitrary large rank is allowed.

- However, string compactifications can realize only QFTs with $r_G \leq 26 - d$.

d	r_g	Compactifications
9	17	Heterotic on S^1
	9	CHL string
	1	M-theory on Klein bottle, Type IIB on DP bg
8	18	Heterotic on T^2
	10	CHL on S^1
	2	9d, $r_g = 1$ on S^1
7	19	Heterotic on T^3
	11	CHL on T^2
	7	F-theory on $K3 \times S^1/\mathbb{Z}_3$
	5	F-theory on $K3 \times S^1/\mathbb{Z}_4$
	3	F-theory on $K3 \times S^1/\mathbb{Z}_{5,6}$ or $T^4 \times S^1/\mathbb{Z}_{3,4,5}$

[Chaudhuri, Hockney, Lykken 95]
[Dabholkar, Park 96],
[de Boer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi 01],
[Aharony, Komargodski, Patir 07],
….

String probes

- Consider again BPS string probes preserving (0,8) SUSY for 2-form B_2 .

- 2-form B_2 transforms under gauge and local Lorentz transformation as

$$\delta_{\Lambda_i,\Theta} B_2 = a\,\mathrm{tr}\Theta R - \frac{1}{4}\sum_i \Omega^{ij}\mathrm{Tr}\Lambda_i F_i$$

not determined due to lack of anomalies

$\Omega^{ij}\ :\ SO(r_G, 10-d)$ invariant metric with $\mathrm{sig}(\Omega) = (r_G, 10-d)$

- Anomaly inflow from the bulk gravity theory predicts that the 2d worldsheet CFT on a single string has anomalies

$$I_4 = \frac{a}{2}p_1(T_2) - a\,c_2(SO(d-2)) + \frac{1}{4}\sum_i \Omega^{ij}\mathrm{Tr}F_i F_j \qquad a \in \mathbb{Z}$$

Central charges of 2d worldsheet CFT

- Central charges of the worldsheet CFT are determined by anomalies as

$$c_L = 24a \ , \quad c_R = 12a$$

- (0,8) center of mass contribution : $(c_L^{\rm com}, c_R^{\rm com}) = (d-2, 12)$

- Also, 't Hooft anomaly $k_{ij} = \Omega_{ij}$ with signature $\mathrm{sig}(\Omega) = (r_G, 10-d)$ implies that current algebras for every bulk gauge group should be realized in the left moving sector. They contribute to the left-moving central charge by $c_G = \dfrac{k \cdot \mathrm{dim}G}{k+h^\vee}$ (which is bigger than the rank of gauge group r_G).

- We therefore find the following unitarity bound from a (0,8) string

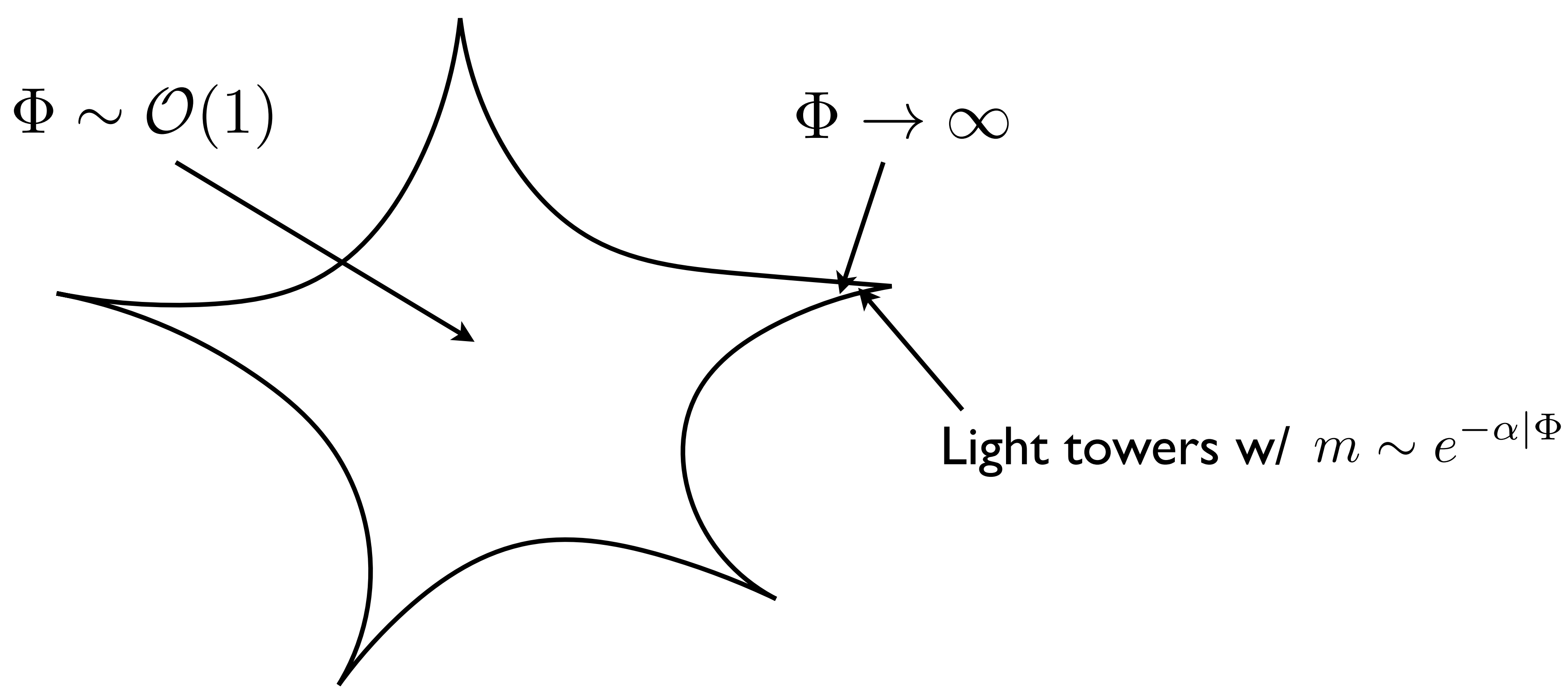
$$r_G \leq \hat{c}_L = 24a + 2 - d$$

- We still need to determine the anomaly coefficient 'a'.

Distance conjecture

- We will now use "Distance conjecture" to show that 'a=1 or 0'.

- Distance conjecture claims that at infinite distance in scalar moduli space, infinite tower of light states must emerge. [Ooguri,Vafa 06]



- This in fact predicts a new weakly-coupled description whose basic modes include this tower of light states.

Distance conjecture and T-duality

- We shall now consider a compactification $d \to d-1$ along a circle with radius R.

- In the limit $R \to 0$, as predicted by distance conjecture, winding string states around the circle give infinite light states with mass

$$m_w \sim nR \quad \to \quad 0$$

- Moreover, distance conjecture together with non-renormalization of the moduli space due to 16 SUSY implies that these light winding string states should be identified with KK momentum states of a graviton in the dual theory.

T-duality : Winding strings $\longleftrightarrow$ KK momentum states in dual d-dim theory

Bounds on gauge group G

- The spectrum of the winding string should contain in R-sector a ground state with spin $J_R = c_R/6 = 2a$. [Lerche, Vafa, Warner 89]

- This winding string state should be mapped to a graviton state with spin <=2 in the dual theory. Therefore, we find $a \leq 1$.

- When $a = 1$, after removing c.o.m contributions, the central charges of the (0,8) worldsheet CFT are $(\hat{c}_L, \hat{c}_R) = (26 - d, 0)$.

- Unitarity of the worldsheet CFT leads to a bound on the rank of the bulk gauge group:

$$r_G \leq \hat{c}_L = 26 - d$$ (agrees with the bound in string compactification)

- When $a = 0$, the worldsheet CFT has enhanced (8,8) SUSY and the same analysis leads to a bound $r_G \leq 10 - d$.

Swampland with 16 SUSY

- We thus claim that supergravity theories preserving 16 SUSY in d-dimensions with $r_G > 26 - d$ belong to the Swampland!
  Moreover, this bound agrees with the bound from string compactifications.

- For example, 4d N=4 SU(N) super Yang-Mills theory cannot consistently couple to <u>4d gravity</u> if N > 23.

- This doesn't mean that a consistent quantum gravity cannot engineer 4d N=4 SU(N) SYM with N > 23. As we know, they are realized as QFTs on N D3-branes in Type IIB string theory. However, the SYM on D3-branes couple to 10d gravity, not 4d gravity.

- Our bound is for QFTs coupled to gravity in the same dimensions.

6d (1,0) Supergravity

6d N=(1,0) Supergravity

- 6d N=(1,0) supermultiplets

Gravity	$(g_{\mu\nu}, B^{+}_{\mu\nu}, \psi^{-}_{\mu})$
Tensor (T)	$(B^{-}_{\mu\nu}, \phi, \chi^{+})$
Vector (V)	(A_{μ}, λ^{-})
Hyper (H)	$(\varphi, \tilde{\varphi}, \psi^{+})$

- The 1-loop anomalies from chiral fields can be cancelled by the Green-Schwarz-Segnotti mechanism only if they factorize as

[Green, Schwarz 1984], [Sagnotti 1992]

$$I_8^{1-loop} = \frac{1}{2}\Omega_{\alpha\beta}X_4^{\alpha}X_4^{\beta} \ ,$$

$$X_4^{\alpha} = \frac{1}{2}a^{\alpha}\mathrm{tr}R^2 + \frac{1}{4}\sum_i b_i^{\alpha}\frac{2}{\lambda_i}\mathrm{tr}F_i^2$$

$\Omega_{\alpha\beta}$: symmetric bilinear form

a^{α}, b_i^{α} : vectors in $\mathbb{R}^{1,T}$

$\alpha, \beta = 1, 2, \cdots, T+1$

$\lambda_{SU(N)} = 1, \lambda_{E_8} = 60, \cdots$

- In addition, the charge lattice Γ of T+1 2-form tensor fields $B_{\mu\nu}$ should be unimodular.

[Kumar, Morrison, Taylor 10], [Seiberg, Taylor 11]

6d N=(1,0) Supergravity

There are infinite families of anomaly free N=(1,0) 6d supergravities.

[Kumar, Morrison, Taylor 2010], …

Ex1) $T = 9\,,\ G = SU(N) \times SU(N)$ with 2 bi-fundamental hypers is anomaly free for arbitrary N, while UV completion is known for $N \leq 8$.

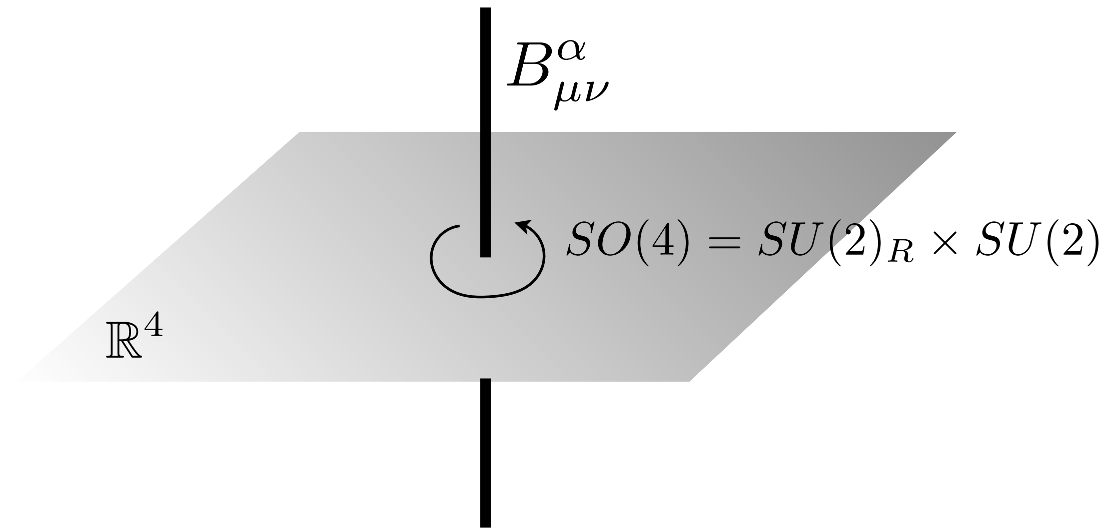
[Dabholkar, Park 1996]

Ex2) $T = 8k + 9\,,\ G = (E_8)^k$ theory is anomaly free for arbitrary k , while UV completion is known only for $k = 1, 2$.

[Seiberg, Witten 1996]

- 6d N=(1,0) supergravity theories can be engineered in F-theory compactified on elliptic Calabi-Yau threefolds, but the number of such theories is finite since the number of elliptic CY3 is finite.

Strings in 6d (1,0) Supergravity

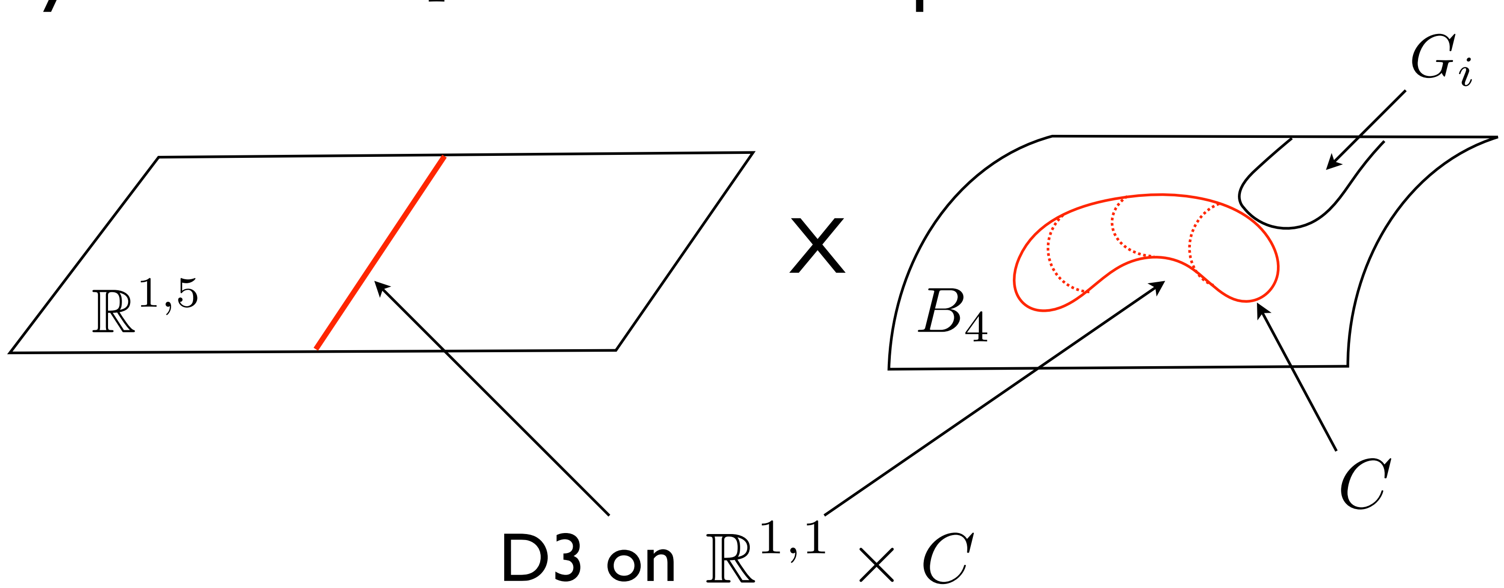


Let us consider BPS string probes on the tensor moduli space.

The 2d worldsheet theory flows to N=(0,4) SCFT in IR.

- Symmetry : $SU(2)_R \times SU(2)_l \times G$ where $G \subset G_{\text{bulk}}$
- $SU(2)_R$ is R-symmetry of IR (0,4) super-algebra.

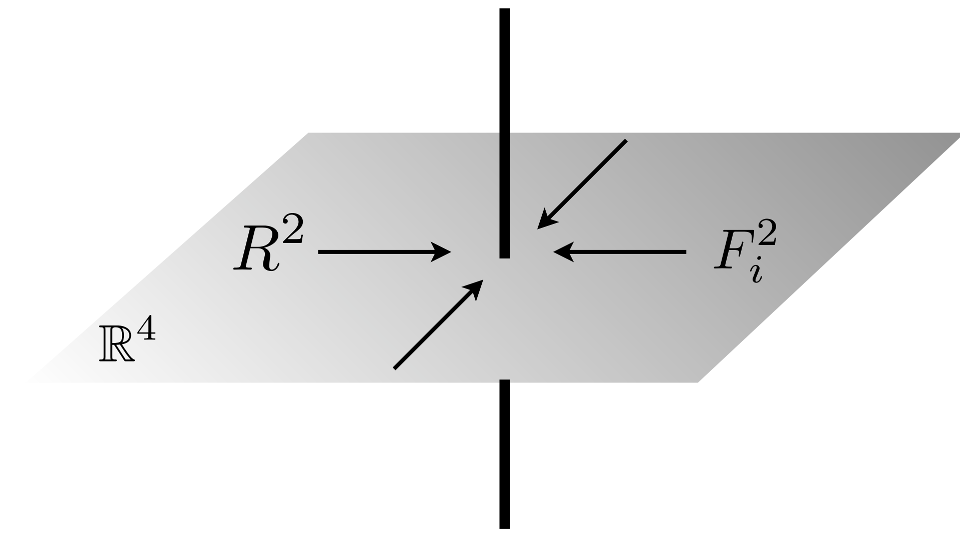
In Type IIB (or F-theory), strings come from D3-branes wrapped on a holomorphic 2-cycle $C \in B_4$ base in an elliptic CY3 X.



Worldsheet Anomalies

- Anomalies of the worldsheet theories can be computed by anomaly inflow.

[Bergman, Harvey 2004], [H-C. Kim, S. Kim, Park 2016], [Shimizu, Tachikawa 2016]



- Central charges of the worldsheet CFT in 6d supergravity theory:
  - Central charges : $\hat{c}_L = 3Q\cdot Q - 9Q\cdot a + 2\ , \quad \hat{c}_R = 3Q\cdot Q - 3Q\cdot a$
  - Levels of $SU(2)_l \times \prod_i G_i$: $\quad k_l = \frac{1}{2}(Q\cdot Q + Q\cdot a + 2)\ , \quad k_i = Q\cdot b_i$

- For a string with $c_L, c_R, k_l, k_i \geq 0$, which we call 'supergravity string', the central charges of the worldsheet CFT should satisfy the inequality:

$$\frac{3k_l}{k_l+2} + \sum_i \frac{k_i\cdot \mathrm{dim}G_i}{k_i + h_i^\vee} \leq \hat{c}_L$$

Example I

6d supergravity theory coupled to T=9 with SU(N)xSU(N) gauge group and two bi-fundamental hypermultiplets. [Kumar, Morrison, Taylor 2010]

- No anomaly for arbitrary N with

$$\Omega = \mathrm{diag}(+1,(-1)^9)\ ,\quad a=(-3,(+1)^9)\ ,$$
$$b_1=(1,-1,-1,-1,0^6)\ ,\quad b_2=(2,0,0,0,(-1)^6)$$

- String theory realization for $N\leq 8$ by [Dabholkar, Park 1996].

Consider a supergravity string with charge $Q=(1,-1,0,0,-1,0^5)$ that has

$$c_L=8\ ,\quad c_R=0\ ,\quad k_l=0\ ,\quad k_1=0\ ,\quad k_2=1$$

Unitarity bound for this string : $\dfrac{k_i\mathrm{dim}G}{k_i+h^\vee}\leq c_L \ \rightarrow\ \dfrac{N^2-1}{N+1}\leq 8\ \rightarrow\ N\leq 9$

<u>Therefore, the theories with N>9 are in the Swampland.</u>

Example II

6d supergravity theory coupled to T=8k+9 with $(E_8)^k$ gauge group.

- No anomaly for arbitrary k with [Kumar, Morrison, Taylor 2010]

$$\Omega = \mathrm{diag}(1, (-1)^{8k+9}) \ , \quad a = (-3, 1^{8k+9}) \ ,$$
$$b_i = (-1, -1, 0^{4(i-1)}, (-1)^3, -3, 0^{8k+8-4i})$$

- A string of charge $Q = (-q, 0^{8k+9})$ with $q \geq 9$ cannot satisfy the unitary bound

$$\frac{3k_l}{k_l+2} + \sum_{i=1}^{k} \frac{248k_i}{k_i+30} \cancel{\leq} c_L$$

- Therefore, all these theories with above data (Ω, a, b_i) are in the Swampland.

- On the other hand, the theories with k=1,2 have another solution to the anomaly cancellation. These theories are realized in string/M-theory. [Seiberg, Witten 1996]

Summary and Outlook

Summary and Outlook

- The Swampland program aims to find universality properties of consistent quantum gravity theories based on the lessons from string theory.

- Completeness of string spectrum and consistency of what lives on string defects can provide us new Swampland conditions.

- We examined the unitarity relation between central charge and levels of gauge group's current algebras in 2d theories on strings and found new Swampland bounds on rank of gauge groups.

- Supergravities with 8 SUSY in $d < 6$? [Katz, H-C. Kim, Tarazi, Vafa 20]
- Refinement of the rank bound? [Montero, Vafa]
- Other types of branes?