Shlomo S. Razamat (Technion)

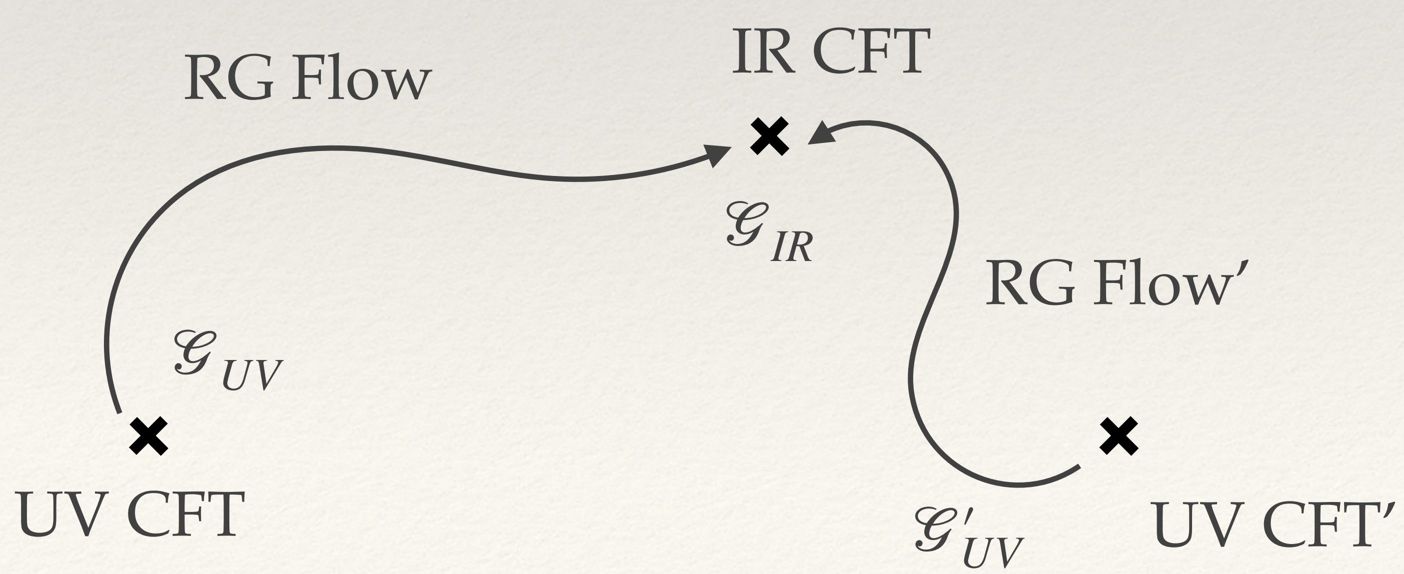
SQCD and Pairs of Pants

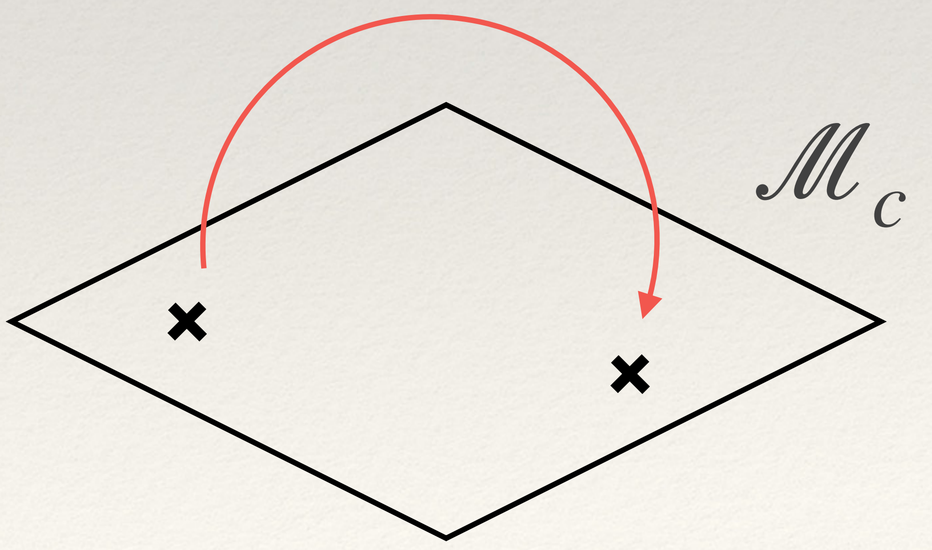
ZOOM, 17/6/2020
Strings and QFTs
for Eurasian time zone

Based on: 2006.03480 with **Evyatar Sabag**

Dynamics of 4d SQFTs

- QFTs in 4d exhibit a wide variety of interesting strong coupling effects
- IR dualities
- Conformal dualities
- Emergence of symmetry in IR
- Is there an organising principle to understand these phenomena?

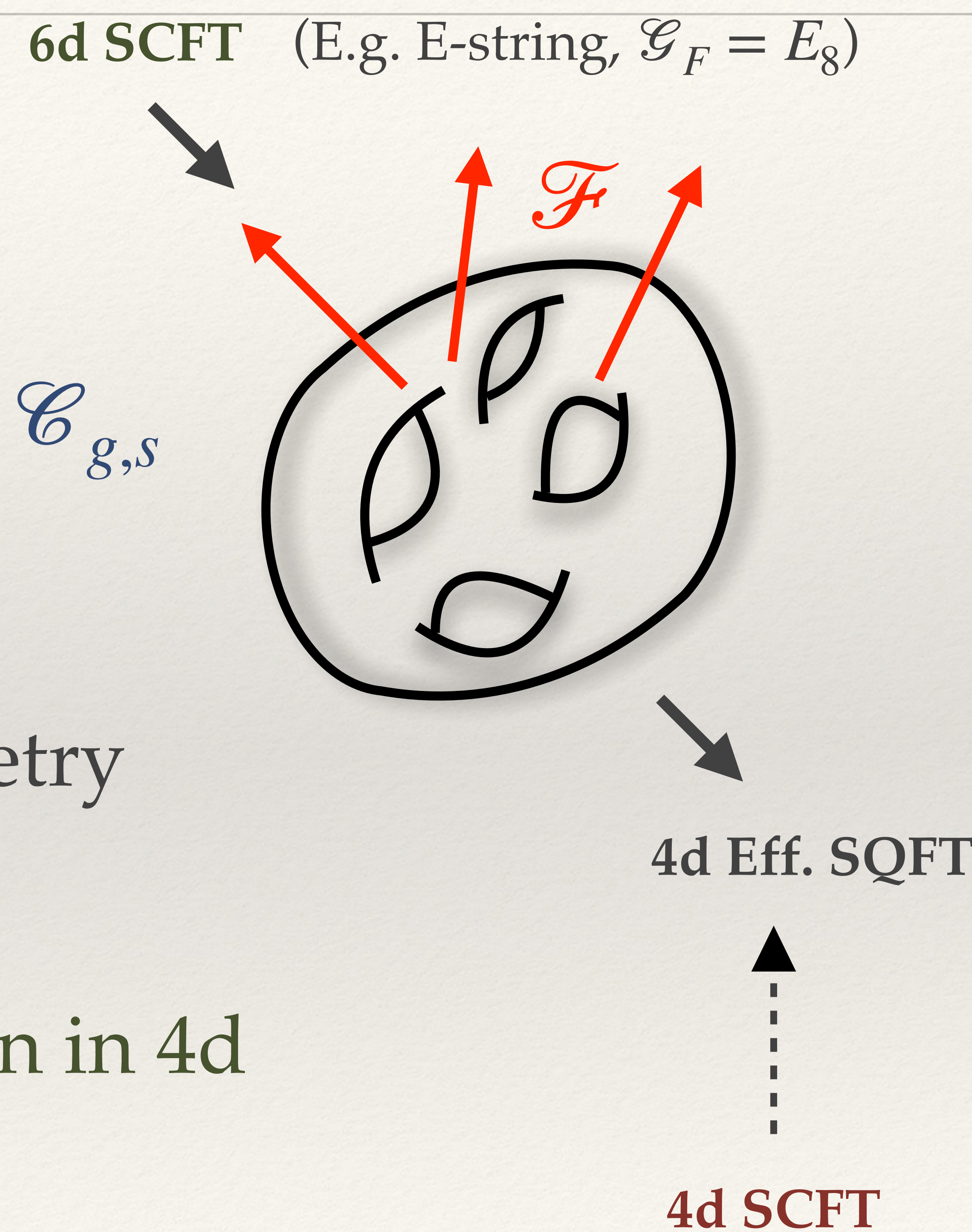




4d SQFTs from 6d SCFTs

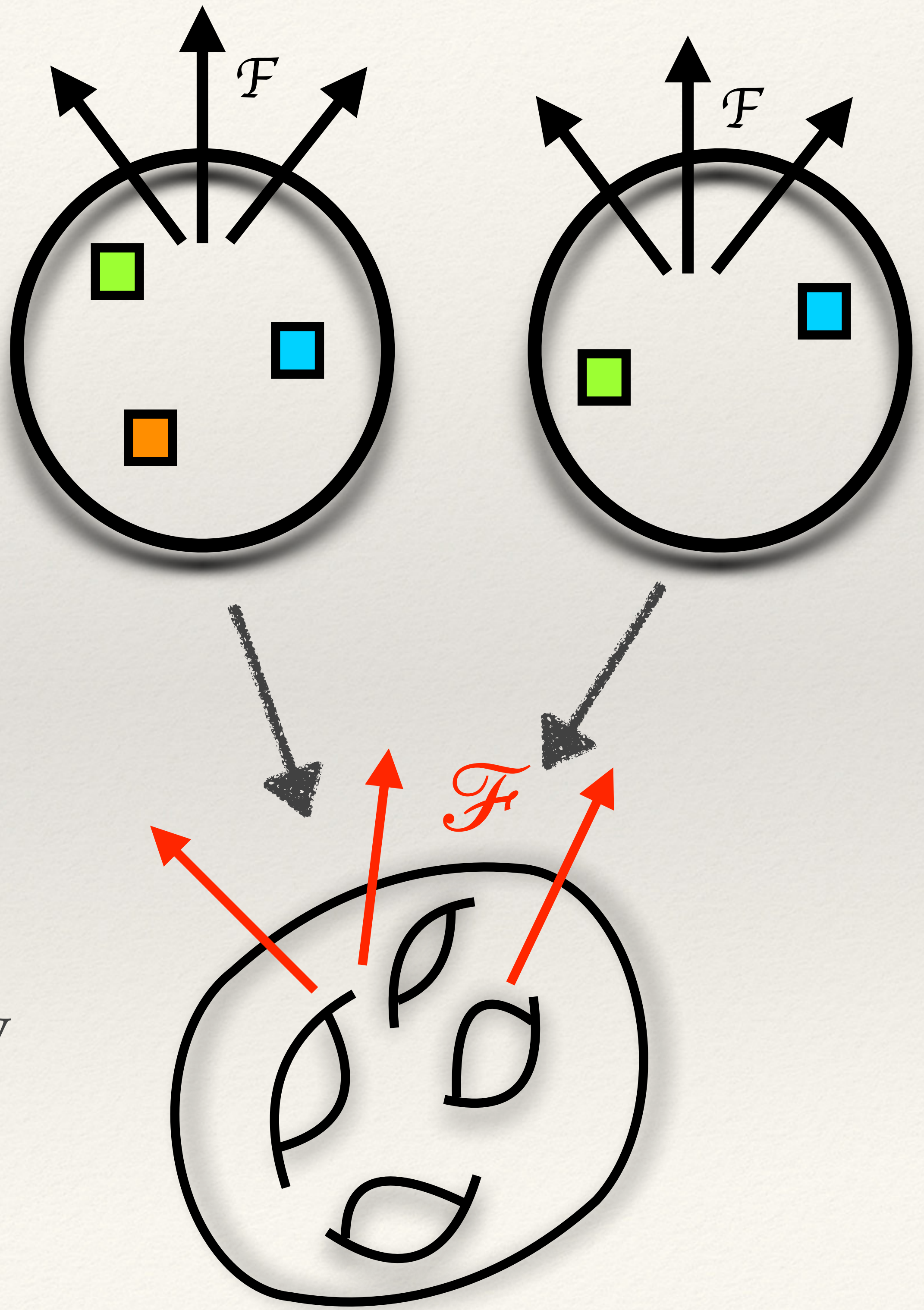
Gaiotto 2009 and many others

- Consider a 6d SCFT
- Put it on (punctured) surface $\mathscr{C}_{g,s}$
- Effective 4d SQFT
- Choices: 6d SCFT and Geometry ($\mathscr{C}_{g,s}$, Flux $\mathscr{F}$, Punctures)
- UV completed in 6d and often in 4d



Dynamics in 4d from 6d

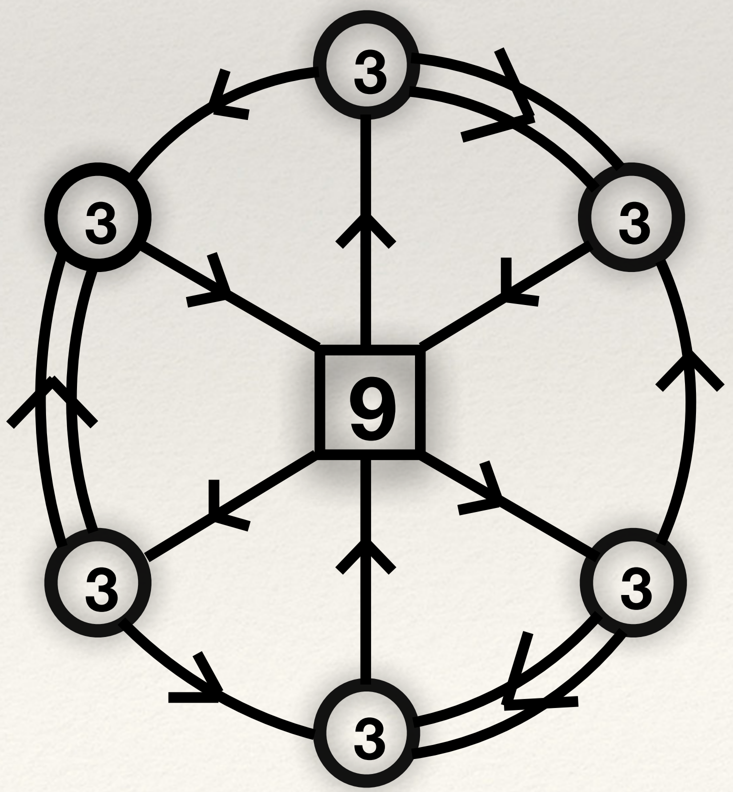
- Dualities and emergent properties in 4d often follow from simple geometric considerations
- One "breaks" the geometry into pieces
- Understands the reduction to 4d of the pieces
- And how to combine the pieces into more complicated geometries
- Properties of combined pieces (equivalence or symmetry) might be trivial geometrically but very surprising field theoretically



A curious observation

SR, Zafrir 2019

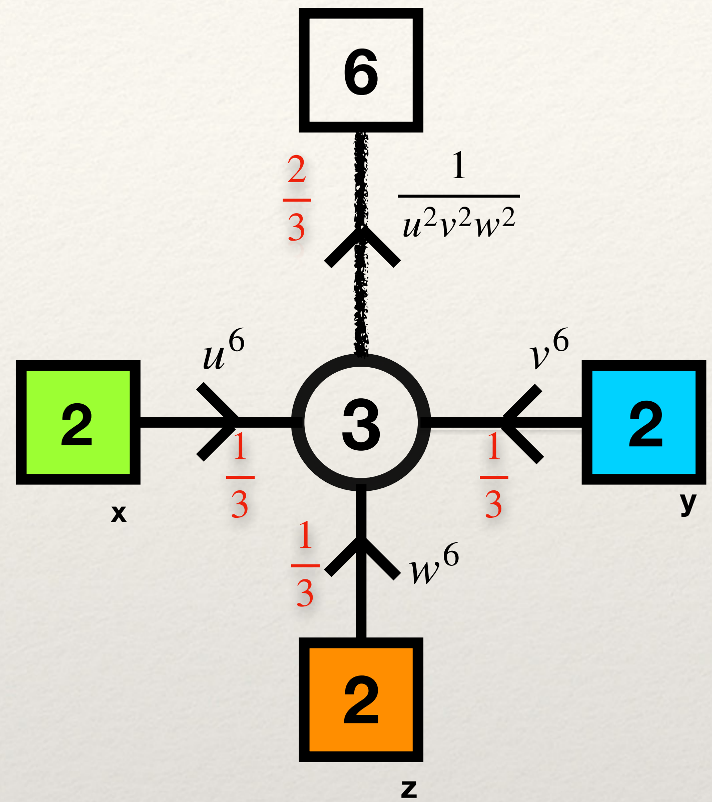
- Compactify E-string on genus $g > 1$ surface $\mathscr{C}_g$
- Anomaly in **4d**: $I_6 = \int_{\mathscr{C}_g} I_8 \;\rightarrow\; a = \frac{75}{16}(g-1), \quad c = \frac{43}{8}(g-1)$
- Assume in 4d described by **Conformal** Gauge theory
- $(a = \frac{9\mathrm{Tr}R^3 - 3\mathrm{Tr}R}{32}, c = \frac{9\mathrm{Tr}R^3 - 5\mathrm{Tr}R}{32}, \; R[Matter] = \frac{2}{3}, \beta_{\mathscr{G},W} = 0)$
- $\rightarrow \quad \dim \mathscr{G} = 16\,(g-1)\,, \quad \dim \mathscr{R} = 81\,(g-1)$
- **Fits a circular $\mathcal{N} = 1$ quiver with** $\mathscr{G} = SU(3)^{2g-2}$
- Cartan of $SU(9) \quad \rightarrow \quad E_8$
- $\dim \mathscr{M}_{conf.} = (3g-3) + (g-1)\,\mathbf{248}$

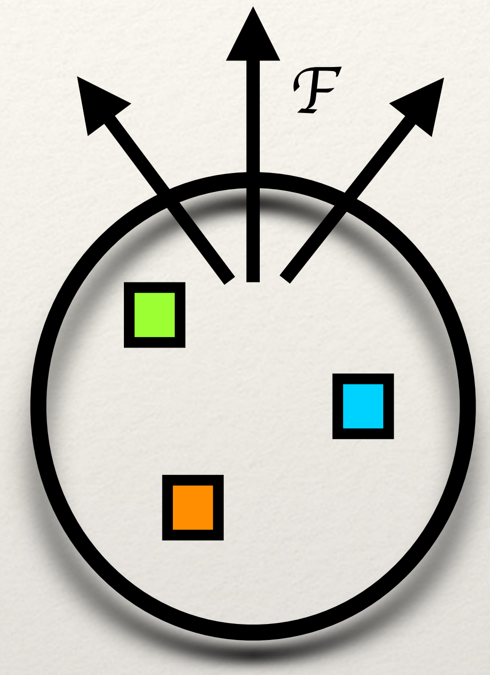


WHY?

E-string trinion

- $SU(3)\ N_f = 6$ SQCD
- $SU(6) \times SU(6) \times U(1)_B \ \rightarrow$
- punct: $SU(2) \times SU(2) \times SU(2)$
- $SU(6) \times U(1)^3 \subset E_8$





``Moment Map" Operators:

(Six Mesons, Two Baryons, R-charge 1)

$$M_u : \mathbf{2}_x \otimes \left(\mathbf{6}_{\frac{u^4}{v^2w^2}} \oplus \mathbf{1}_{u^6v^{12}} \oplus \mathbf{1}_{u^6w^{12}} \right)$$

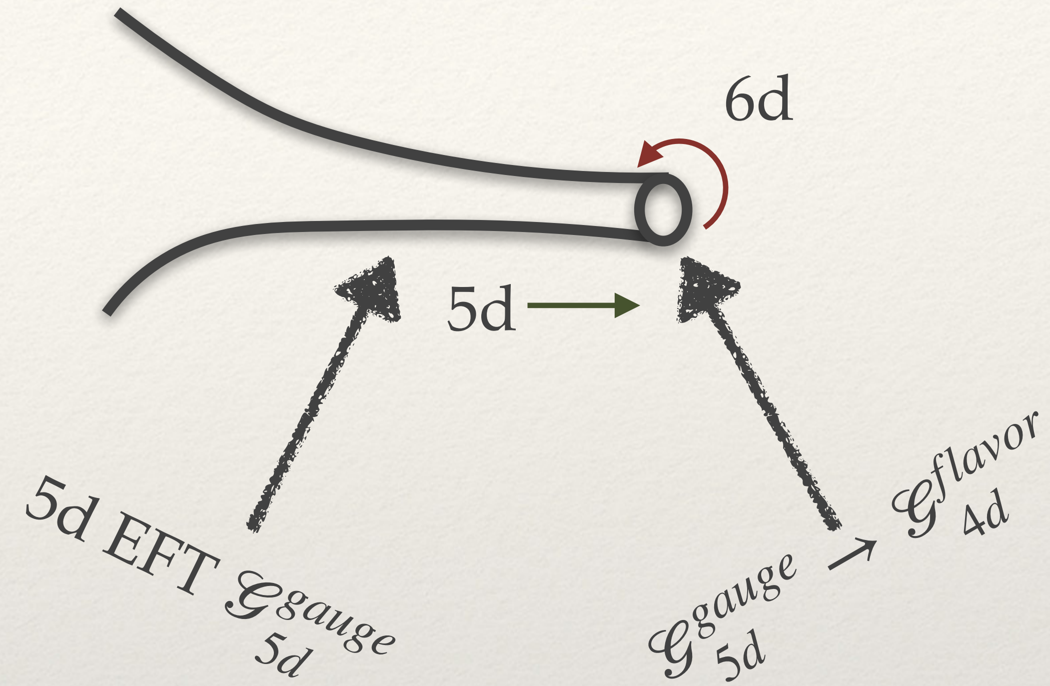
$$M_v : \mathbf{2}_y \otimes \left(\mathbf{6}_{\frac{v^4}{u^2w^2}} \oplus \mathbf{1}_{v^6u^{12}} \oplus \mathbf{1}_{v^6w^{12}} \right)$$

$$M_w : \mathbf{2}_z \otimes \left(\mathbf{6}_{\frac{w^4}{u^2v^2}} \oplus \mathbf{1}_{w^6u^{12}} \oplus \mathbf{1}_{w^6v^{12}} \right)$$

Punctures and 5d

- Compactifying on a surface with punctures we can elongate the region near the puncture into a long cylinder with a boundary
- On a cylinder, with suitable holonomies, get sometimes effective description as a 5d gauge theory
- Natural boundary conditions freezing the 5d gauge group and makes it 4d global symmetry
- The matter fields with Neumann boundary condition give a natural set of operators charged under this symmetry

Kim, SR, Vafa, Zafrir 2017



A_{N-1}(2,0) 5d EFT:

$$\mathcal{G}^{gauge}_{5d} = SU(N)\,, \ \oplus \text{Adj.}$$

Moment maps: 1 Adj χ-op.

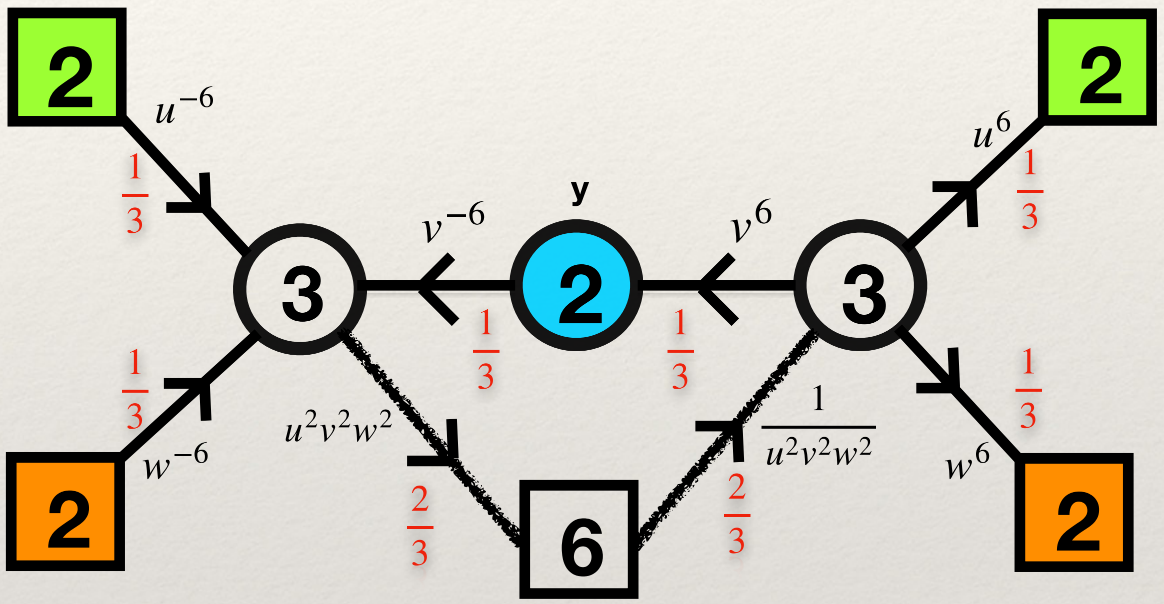
E-string 5d EFT: Seiberg 1996

$$\mathcal{G}^{gauge}_{5d} = SU(2)\,, N_f = 8$$

"Moment maps:" 8 $\square$ χ-op.

S-gluing

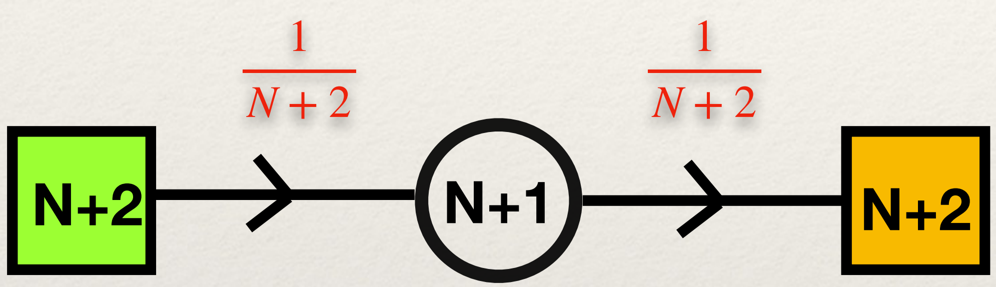
- Take two punctures of the same type (same pattern of charges of the moment maps)
- Gauge the diagonal $SU(2)$ puncture symmetry
- Turn on superpotential,
- $$W = \sum_{i=1}^{8} M_i M_i' .$$
- (If glue two same theories the resulting flux is zero)

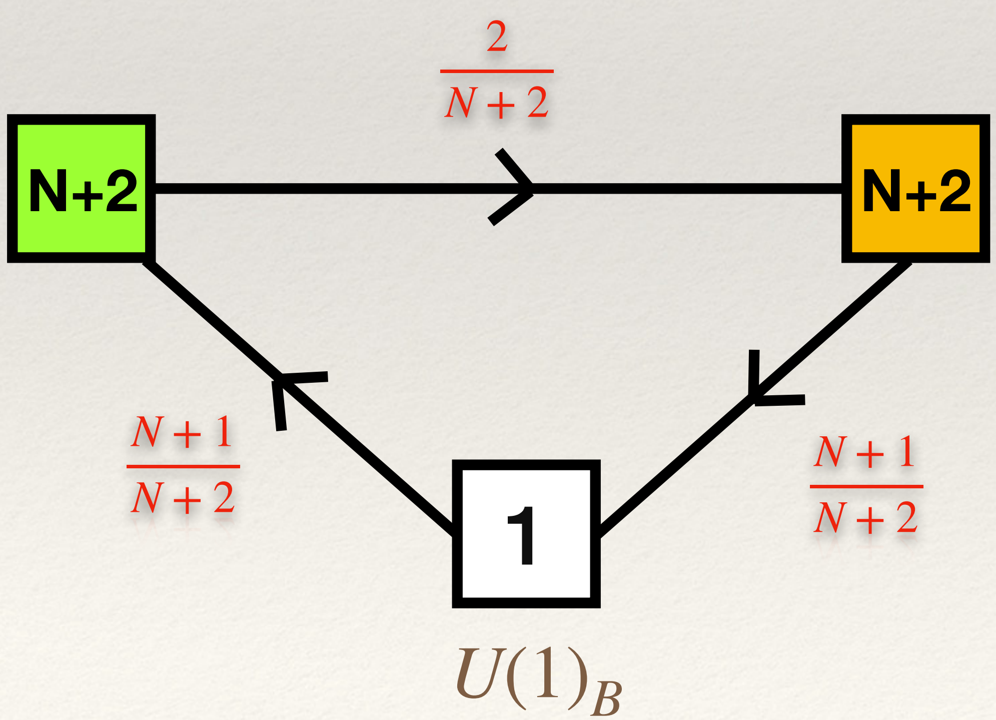


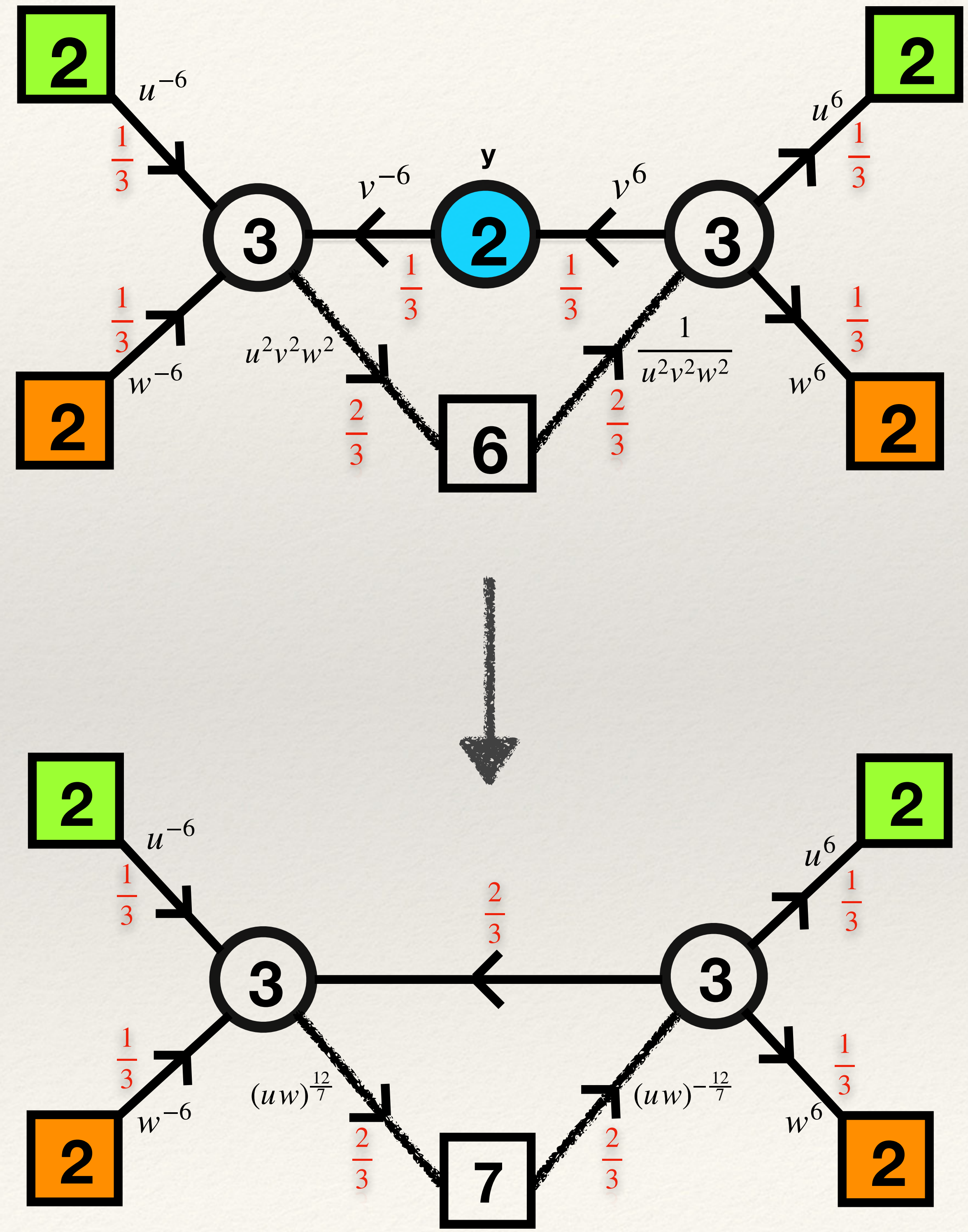
(Interesting dynamics, gauging, quartic and sextic superpotentials (dangerously irrelevant))

S-gluing and Seiberg Duality

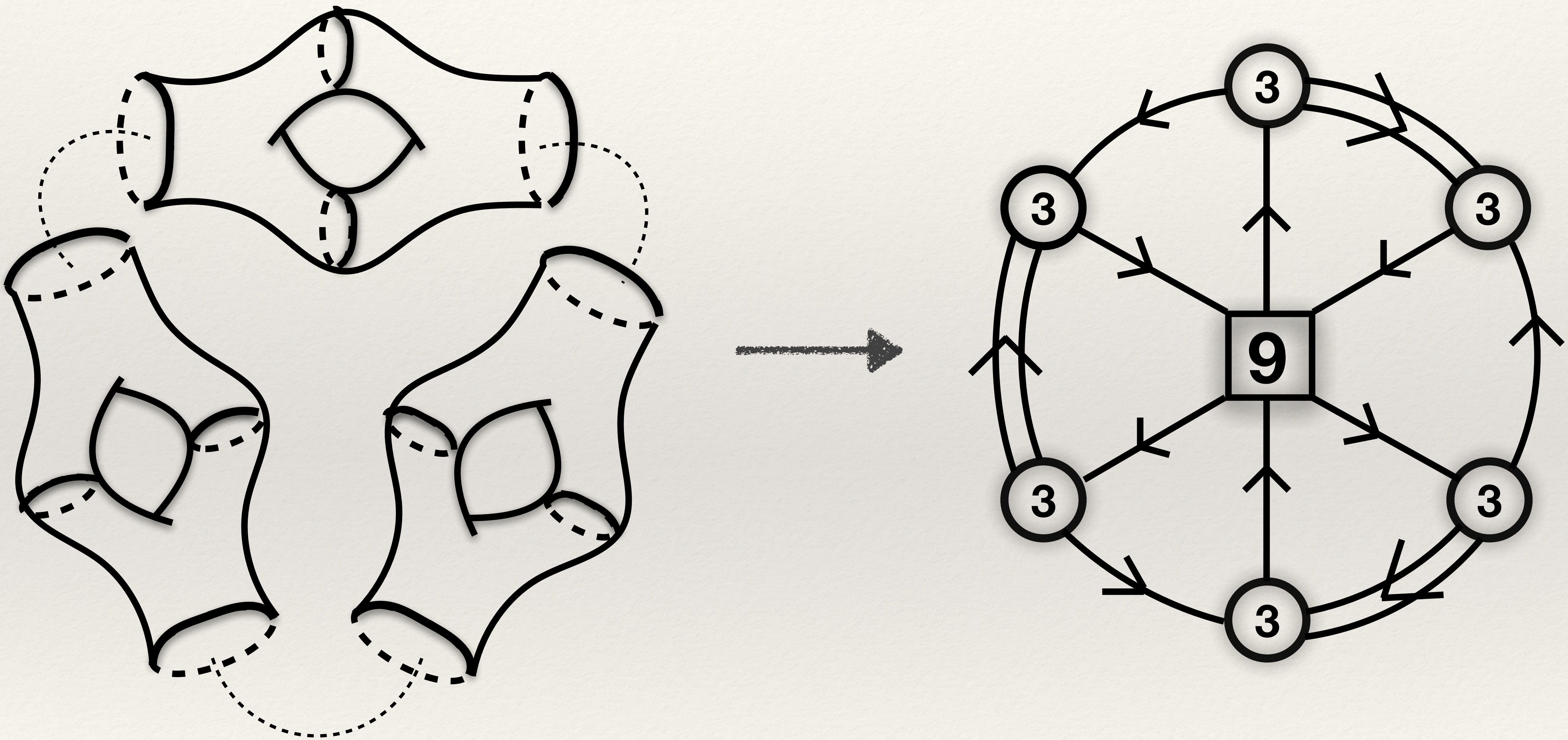
- Use Seiberg duality





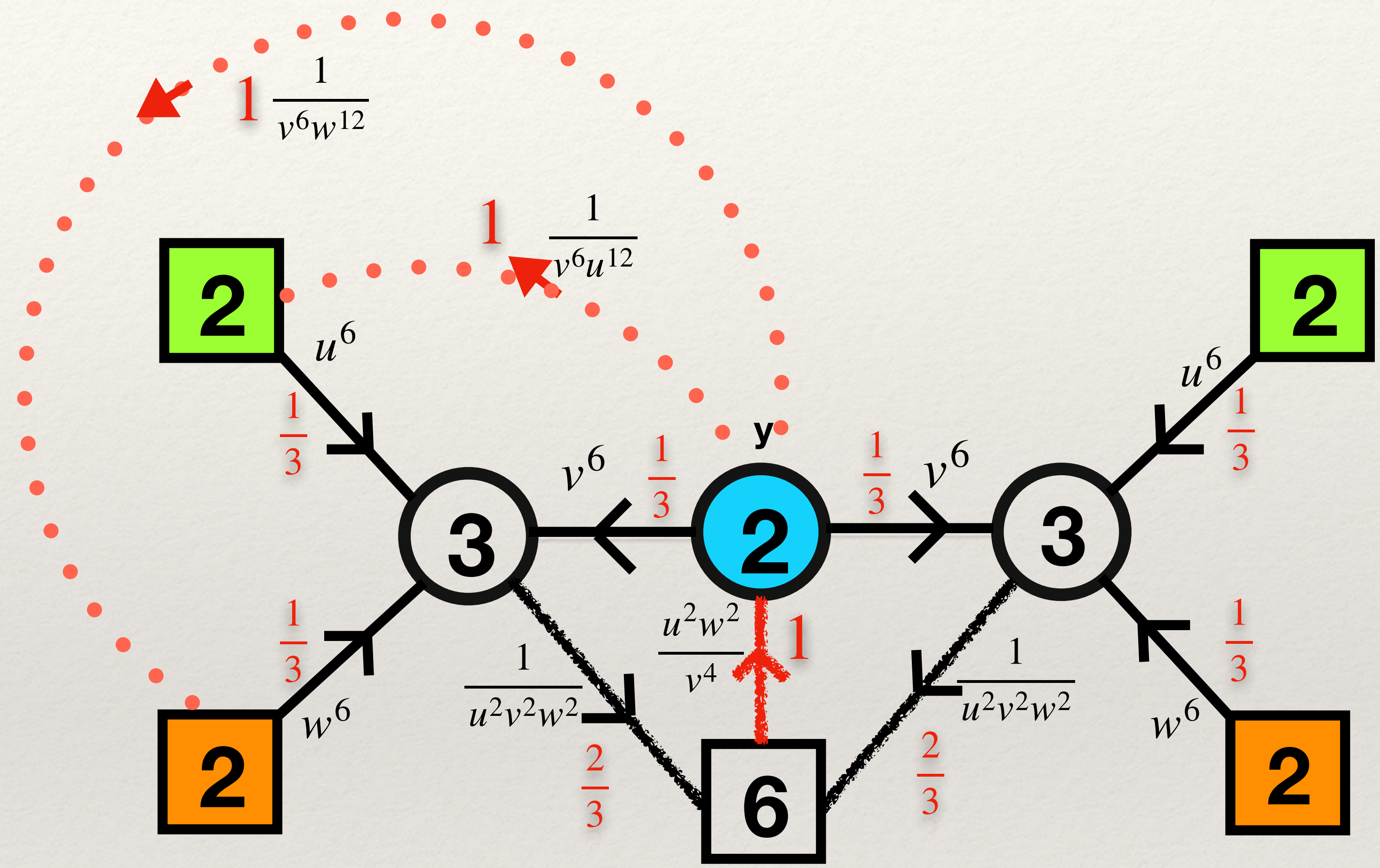


S-gluing into the "Wheel of the Law"



Φ-gluing

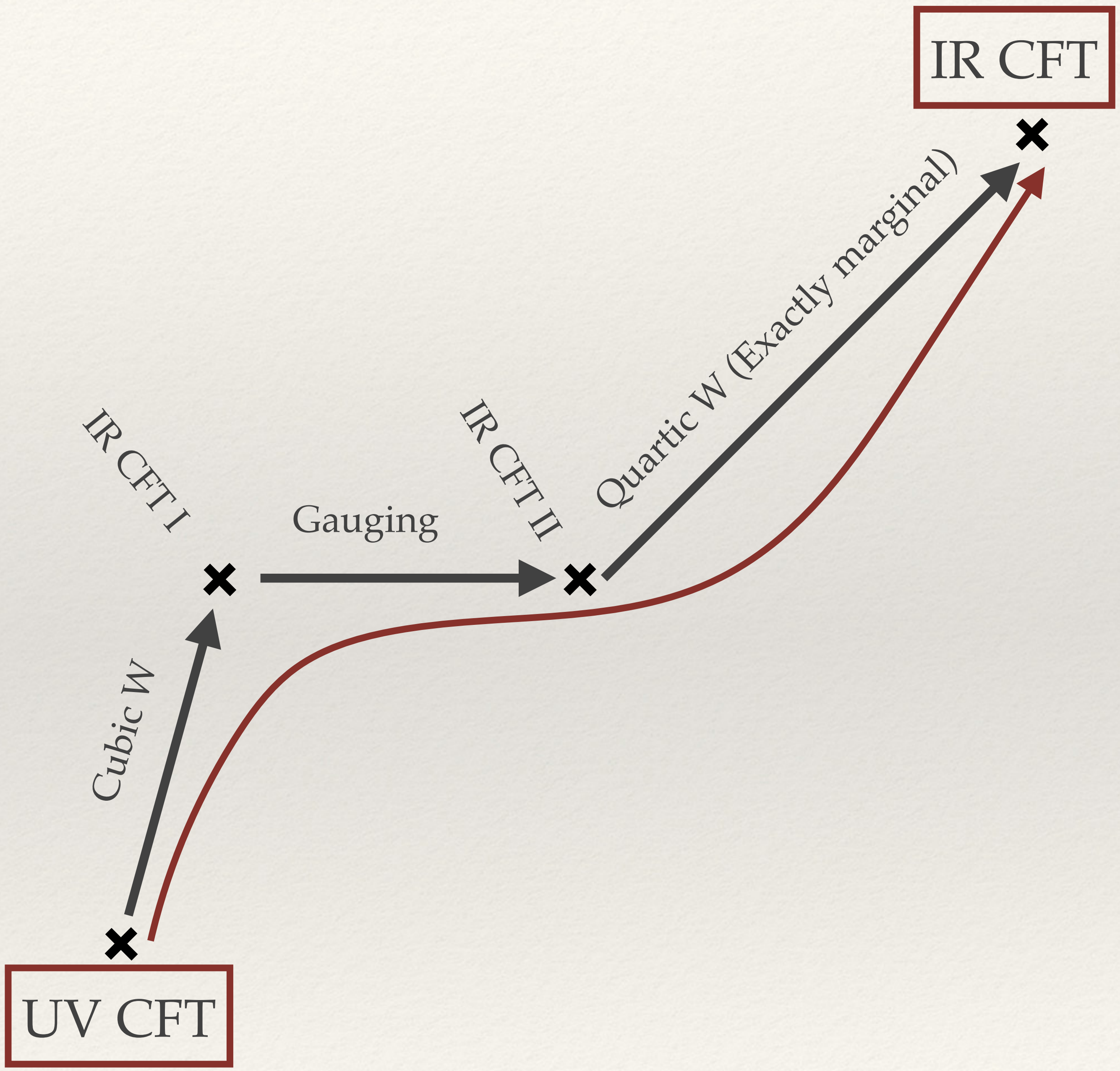
- Take two punctures of the same type (same pattern of charges of the moment maps)
- Gauge the diagonal *SU*(2) puncture symmetry
- Add octet of fundamental fields and turn on superpotential,
- $$W = \sum_{i=1}^{8} \left(M_i \Phi_i - M_i' \Phi_i \right) .$$
- (If glue two same theories the resulting flux is doubled)



(Interesting dynamics, gauging, quartic and cubic superpotentials (dangerously irrelevant))

Dangerously Irrelevant deformations

- In the UV superconformal R-symmetry of the chiral fields of the trinion is $\frac{1}{2}$
- $SU(2)$ gauge group has seven flavors
- Flow in steps
- Step 1: Introduce $\Phi_{3,..,8}$, cubic W and flow
- Step 2: Introduce $\Phi_{1,2}$, Gauging becomes asymptotically free, gauge
- Step 3: Quartic W becomes exactly marginal, turn it on



Φ-gluing and the Flux

- Φ-glue two trinions into a genus two surface
- The flux is twice the flux of the trinion
- Compare anomalies to 6d
- The anomalies match if we assign half a unit of flux to $U(1)$ breaking E_8 to $E_7 \times U(1)$
- The global symmetry of the theory is $E_7 \times U(1)$

$$a = \frac{5\sqrt{5}}{4} + \frac{47}{16}\,, \qquad c = \frac{3\sqrt{5}}{2} + \frac{27}{8}\,.$$

Cartan with the flux

$$\begin{aligned} E_8 \;&\to\; E_7 \times SU(2)_{u^6v^6w^6} \\ &\to\; SU(6) \times SU(3)_{u^8/(w^4v^4),v^8/(w^4u^4)} \times SU(2)_{u^6v^6w^6} \end{aligned}$$

- E.g. superconf. index:

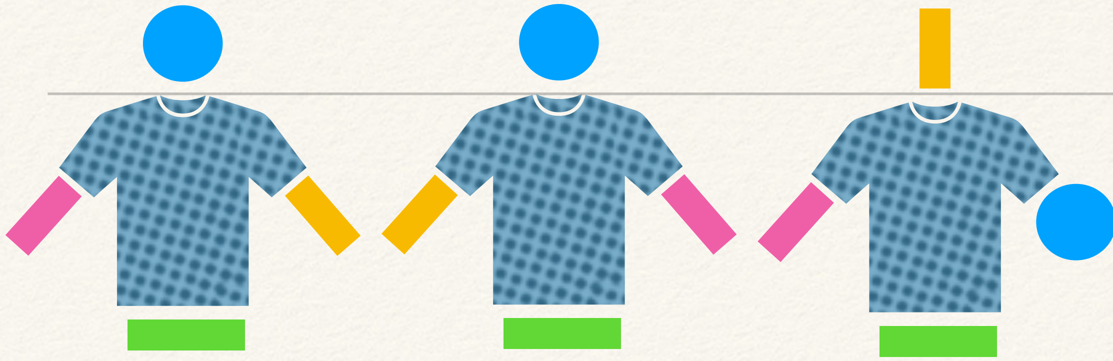
$$1 + qp\left(3 + \left\{1 + \mathbf{133}_{E_7}\right\} + 3\frac{1}{u^{12}v^{12}w^{12}} + 2\frac{1}{u^6v^6w^6}\mathbf{56}_{E_7}\right) + \cdots\,.$$

- General expected structure:

$$1 + qp\left(3g - 3 + (g-1)\left\{1 + \mathbf{133}_{E_7}\right\} + (g-1+2\mathcal{F})\frac{1}{u^{12}v^{12}w^{12}} + (g-1-2\mathcal{F})u^{12}v^{12}w^{12} + (g-1+\mathcal{F})\frac{1}{u^6v^6w^6}\mathbf{56}_{E_7} + (g-1-\mathcal{F})u^6v^6w^6\mathbf{56}_{E_7}\right) + \cdots$$

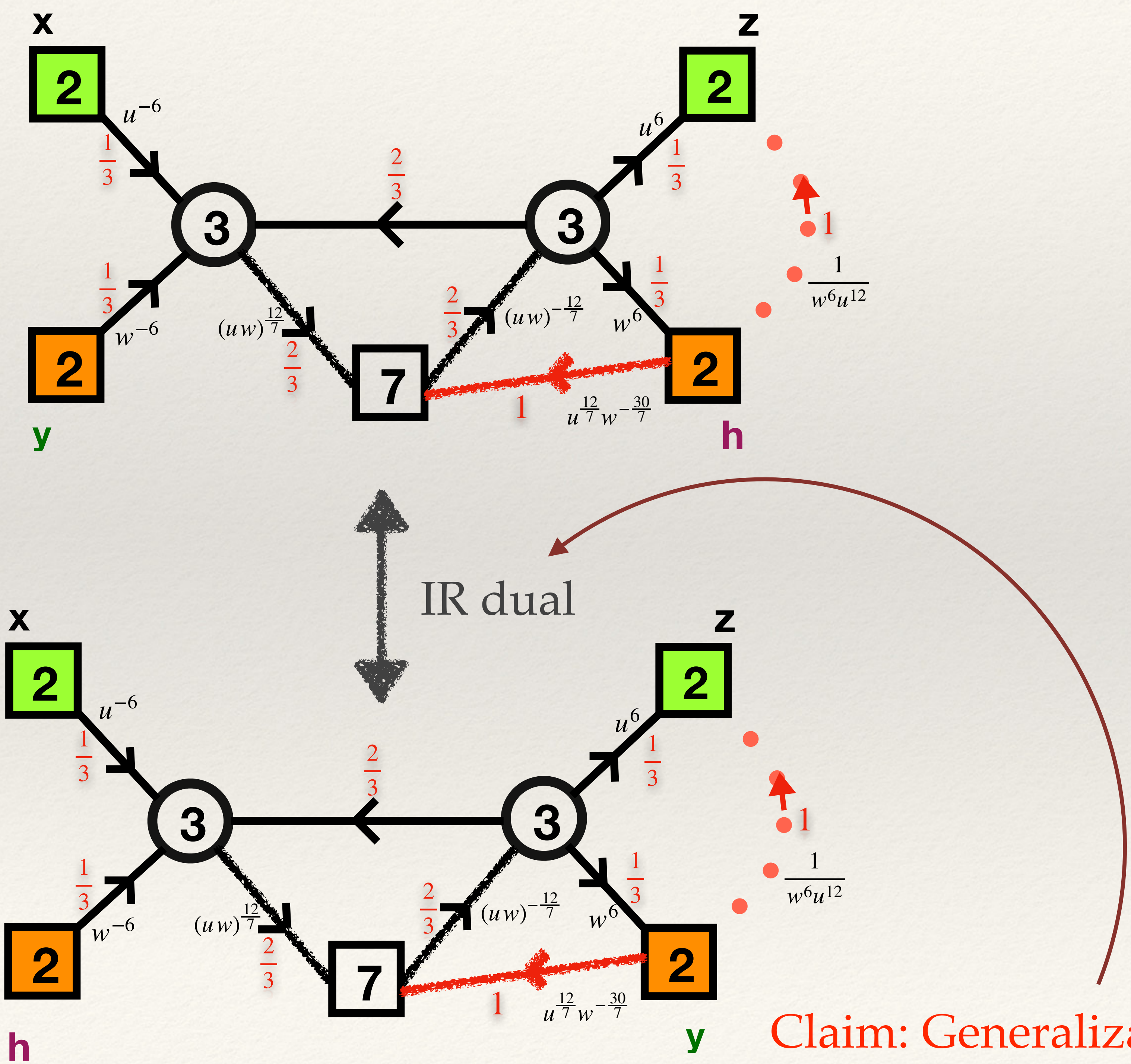
$$g = 2\,, \; \mathcal{F} = 1$$

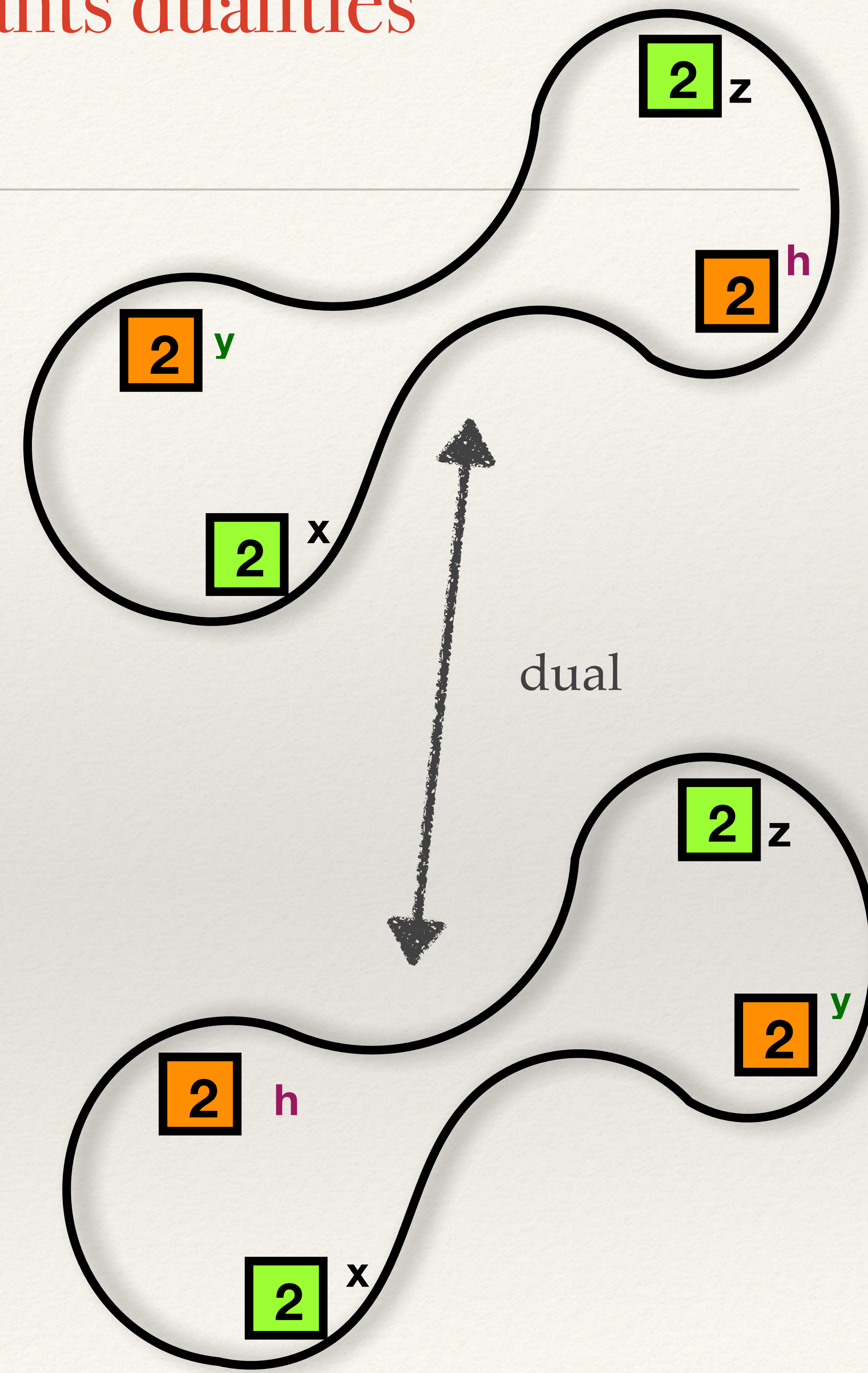
Beem, SR, Zafrir ∞



Pair of pants dualities

- S-gluing example





Claim: Generalization of Intriligator-Pouliot 1995

Vector of Fluxes

- We have **8** $U(1)$ symmetries
- We have **8** moment maps
- Each moment map component charged under different $U(1)$
- Vector of fluxes: $\mathcal{F} = (\mathcal{F}_1, \cdots, \mathcal{F}_8)$
- Trinion: $\mathcal{F}_t = (-1, -1,0,0,0,0,0,0)$
- Preserves $E_7 \times U(1) \subset E_8$
- **Vector of fluxes defined in terms of the moment map symmetries of a given puncture**

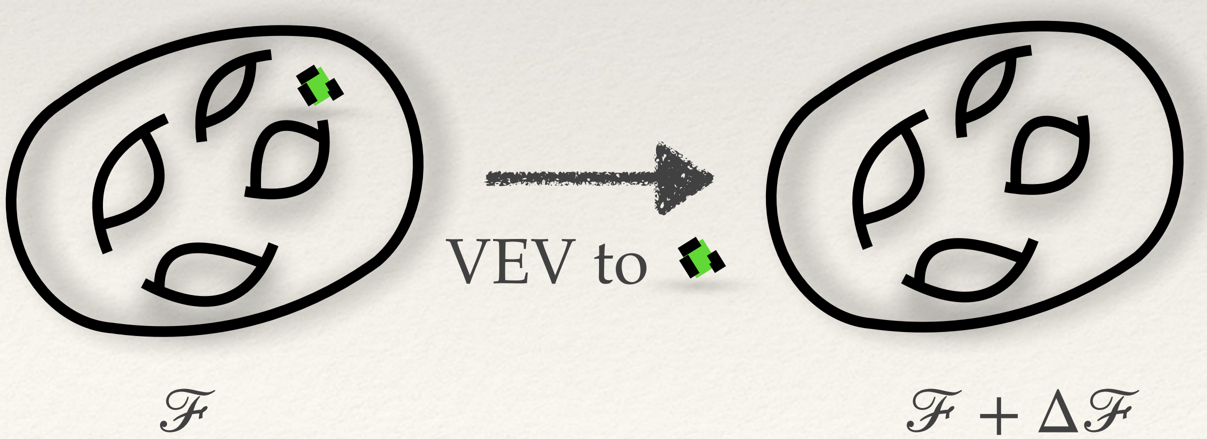
- $SO(16)$ roots: $\mathcal{F} = (1,1,0,0,0,0,0,0)$ + perm. + even # sign flips
- $SO(16)$ spinor weights: $\mathcal{F} = (\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$ + even # sign flips
- $\mathbf{248}_{E_8} = \mathbf{120}_{SO(16)} + \mathbf{128}_{SO(16)}$
- Symmetry preserved by $\mathcal{F}$ spanned by roots and spinorial weights $\perp$ to $\mathcal{F}$
- $\mathcal{F}_t$ orthogonal to $SO(12) \times SU(2)$ roots and 64 spinorial weights
- $\frac{12 \times 11}{2} + 3 + 64 = 133 = \dim E_7$

Flows triggered by VEVs and closing Punctures

- Giving a VEV to one of the moment maps, say M_1^+, breaks $SU(2)$ puncture symmetry
- Adding octet of fields and W:

$$W = \Xi_1 M_1^- + \sum_{i=2}^{8} \Xi_i M_i^+$$

- "Closes" the puncture.
- Vector of fluxes shifted by:
- $\Delta\mathcal{F} = (2,0,0,0,0,0,0,0)$



- Baryonic VEV
- $\mathcal{F}_t + \Delta\mathcal{F} = (1,-1,0,0,0,0,0,0)$
- Preserves $E_7 \times U(1)$
- Mesonic VEV
- $\mathcal{F}_t + \Delta\mathcal{F} = (-1,-1,2,0,0,0,0,0)$
- Preserves $E_6 \times SU(2) \times U(1)$
- The resulting two punctured spheres when glued to tori should exhibit these symmetries

Kim, SR, Vafa, Zafrir 2017

Two punctured spheres and tori from flows

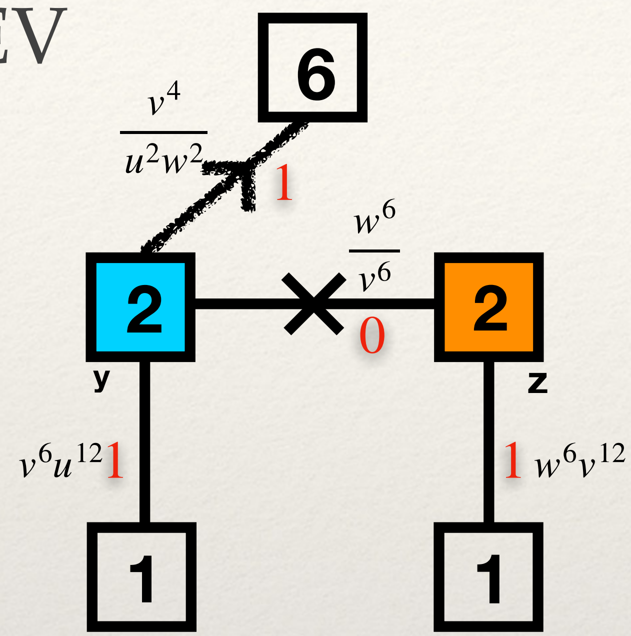
- Baryonic VEV

$(1,-1,0,0,0,0,0,0)$

$(\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$

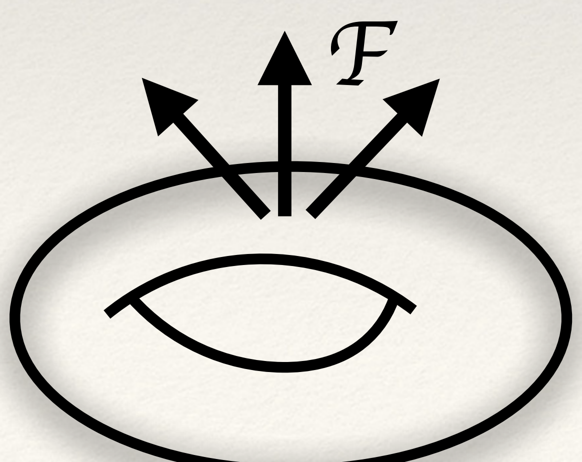
$(-\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$

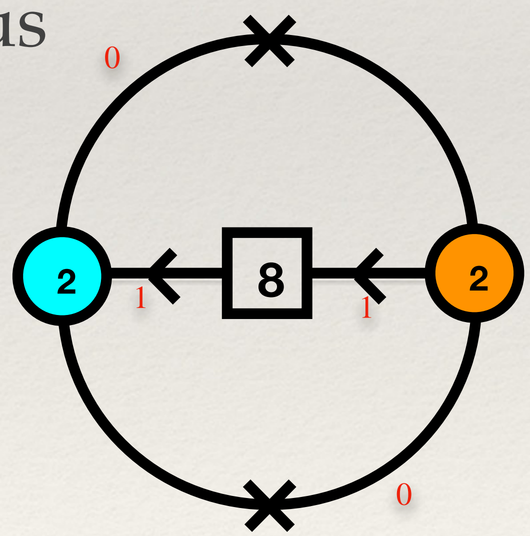
(Related by Weyl)



- Glue two to torus

$E_7 \times U(1)$





- Mesonic VEV

$(-1,-1,2,0,0,0,0,0)$

$(-\frac{1}{2},-\frac{3}{2},\frac{3}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$

$(-\frac{1}{2},-\frac{3}{2},\frac{3}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$

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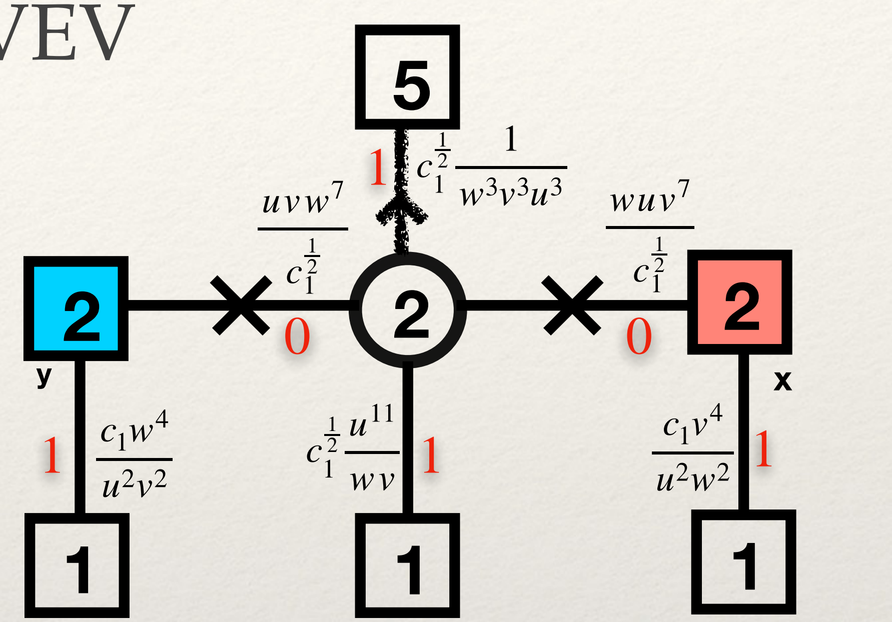
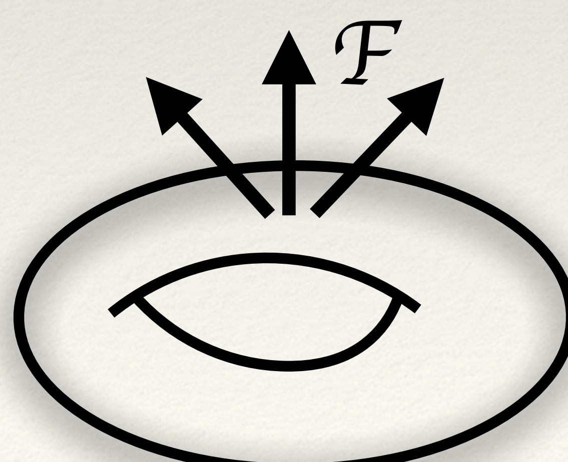
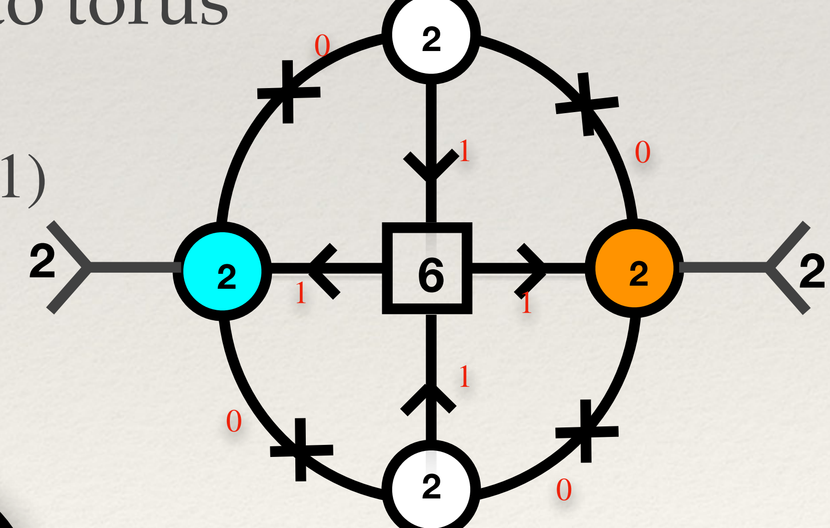


- Glue two to torus

$E_6 \times SU(2) \times U(1)$



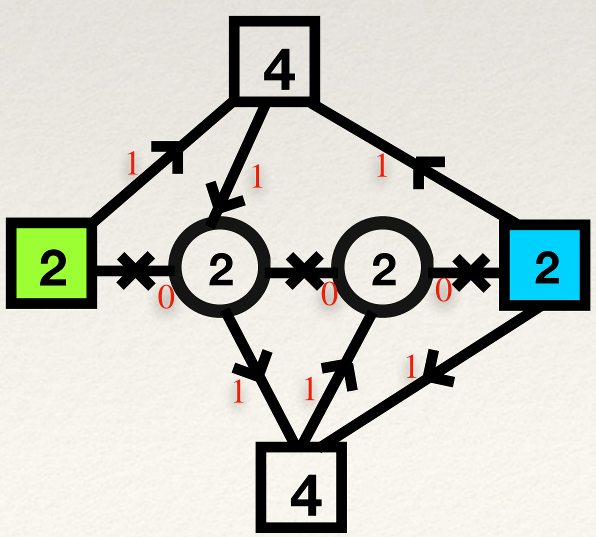


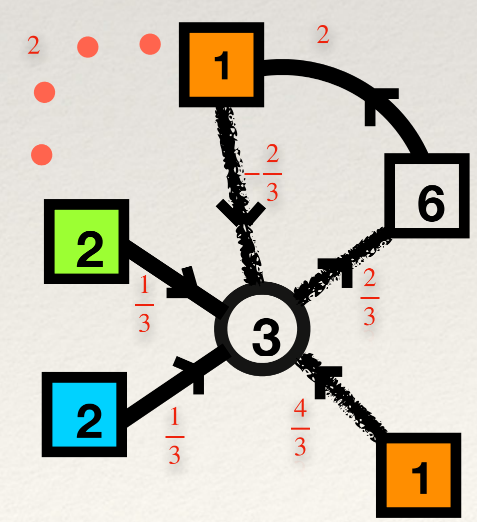
Match Kim, SR, Vafa, Zafrir 2017

Flux decomposition dualities

- Flip one of the mesonic moment maps
- $(-1,-1,-0,0,0,0,0,0)$
- Close the flipped mesonic moment map
- $(-1,-1,2-0,0,0,0,0,0)$
- Same flux as before but field theoretically very different operation, VEV to flip is mass to meson
- Get $SU(3)\ N_f = 5$
- Which is Seiberg dual to $SU(2)\ N_f = 5$ we got before

- Flip one of the baryonic moment maps
- $(1,-1,0,0,0,0,0,0)$
- Close the flipped baryonic moment map
- $(3,-1,0,0,0,0,0,0)$
- Flux preserving $SO(12)\times U(1)^2$, Get $SU(3)\ N_f = 6$ with baryonic superpotential
- Same flux as combination of $3\ E_7$ tubes
- $2\times(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})+(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2})=$
- $(\frac{3}{2},\frac{3}{2},\frac{3}{2},\frac{3}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$



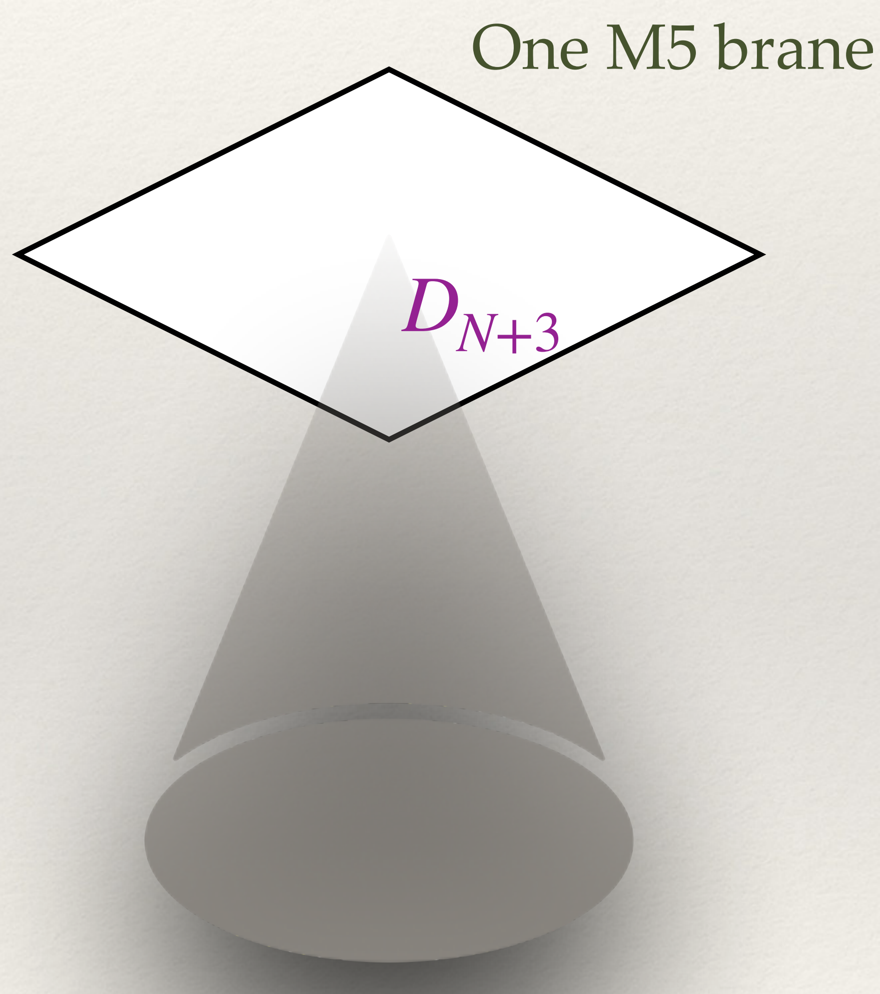


$(-\frac{5}{2},-\frac{3}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$

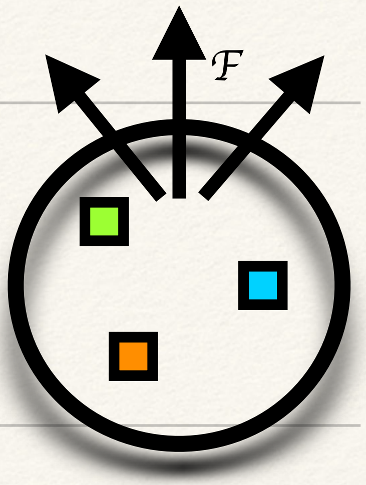
Minimal D conformal matter

- The discussion can be generalised at least in one way
- E-string is the theory residing on a single M5 brane probing D_4 singularity
- Let us consider the theory residing on a single M5 brane probing D_{N+3} singularity
- A non-trivial 6d SCFT, $\mathcal{G}_F = SO(4N+12)$
- *(N>0, Enhances to E_8 for $N=1$)*
- *Minimal (D_{N+3}, D_{N+3}) conformal matter*

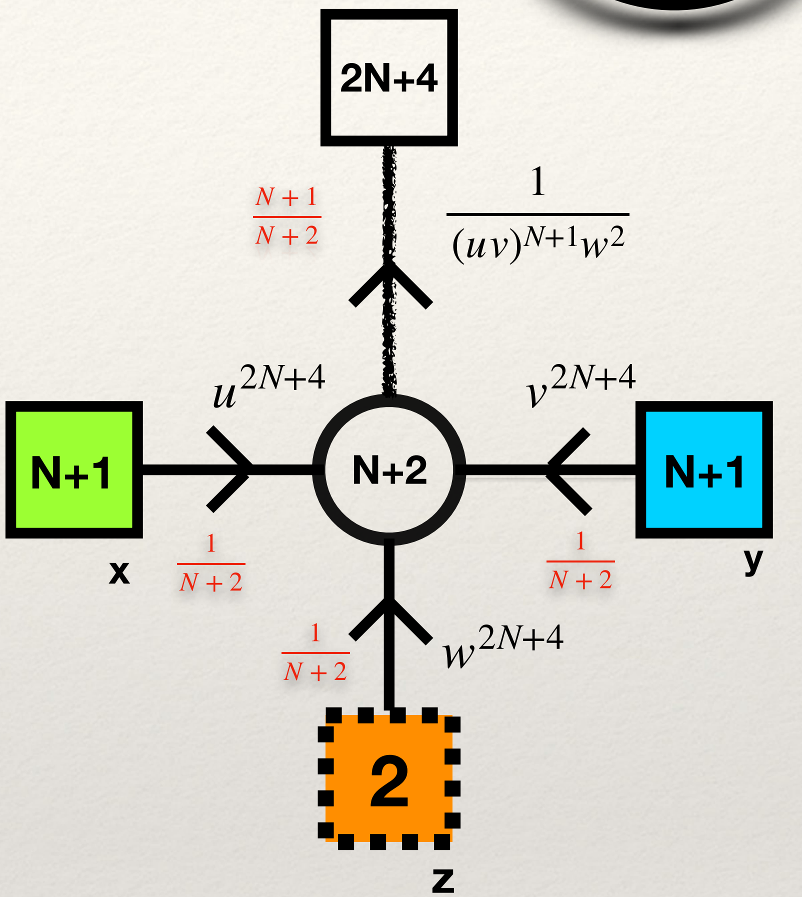
Del Zotto, Heckman, Tomasiello, Vafa 2014



Minimal D conformal matter trinion



- $SU(N+2)\ N_f = 2N+4$ SQCD
- $SU(2N+4)^2 \times U(1)_B\ \to$
- punct: $SU(N+1) \times SU(N+1) \times SU(2)$
- $SU(2N+4) \times U(1)^3 \subset SO(4N+12)$



``Moment Map" Operators:

($2N+4$ Mesons, Two Baryons, R-charge 1)

$$M_u : \mathbf{N+1}^x \otimes \left(\mathbf{2N+4}_{u^{N+3}v^{-(N+1)}w^{-2}} \oplus \mathbf{1}_{(uv^{N+1})^{2N+4}}\right) \oplus \overline{\mathbf{N+1}}^x \otimes \mathbf{1}_{(u^N w^2)^{2N+4}}$$

$$M_v : \mathbf{N+1}^y \otimes \left(\mathbf{2N+4}_{v^{N+3}u^{-(N+1)}w^{-2}} \oplus \mathbf{1}_{(vu^{N+1})^{2N+4}}\right) \oplus \overline{\mathbf{N+1}}^y \otimes \mathbf{1}_{(v^N w^2)^{2N+4}}$$

$$M_w : \quad \mathbf{2}^z \otimes \left(\mathbf{2N+4}_{(uv)^{-(N+1)}w^{2N+2}} \oplus \mathbf{1}_{(wv^{N+1})^{2N+4}} \oplus \mathbf{1}_{(wu^{N+1})^{2N+4}}\right)$$

Punctures and 5d

- Minimal D conformal matter compactified on a circle with certain holonomy, $\mathcal{N}=1$ $SU(N+1)$ with 2N+6 hypers
- In 4d obtain puncture with $SU(N+1)$ global symmetry
- $(2N+6)$ plet of fundamental moment maps
- In fact at least two additional 5d descriptions are known:
- $USp(2N)$ gauge theory and $SU(2)^N$ gauge theory Hayashi, Kim, Lee, Taki, Yagi 2015
- (*For* $N=1$ *all are the same*)
- The additional descriptions will also play a role
- E.g. the minimal $SU(2)$ puncture is a partial closure of $USp(2N)$ puncture

SR, Sabag 2019

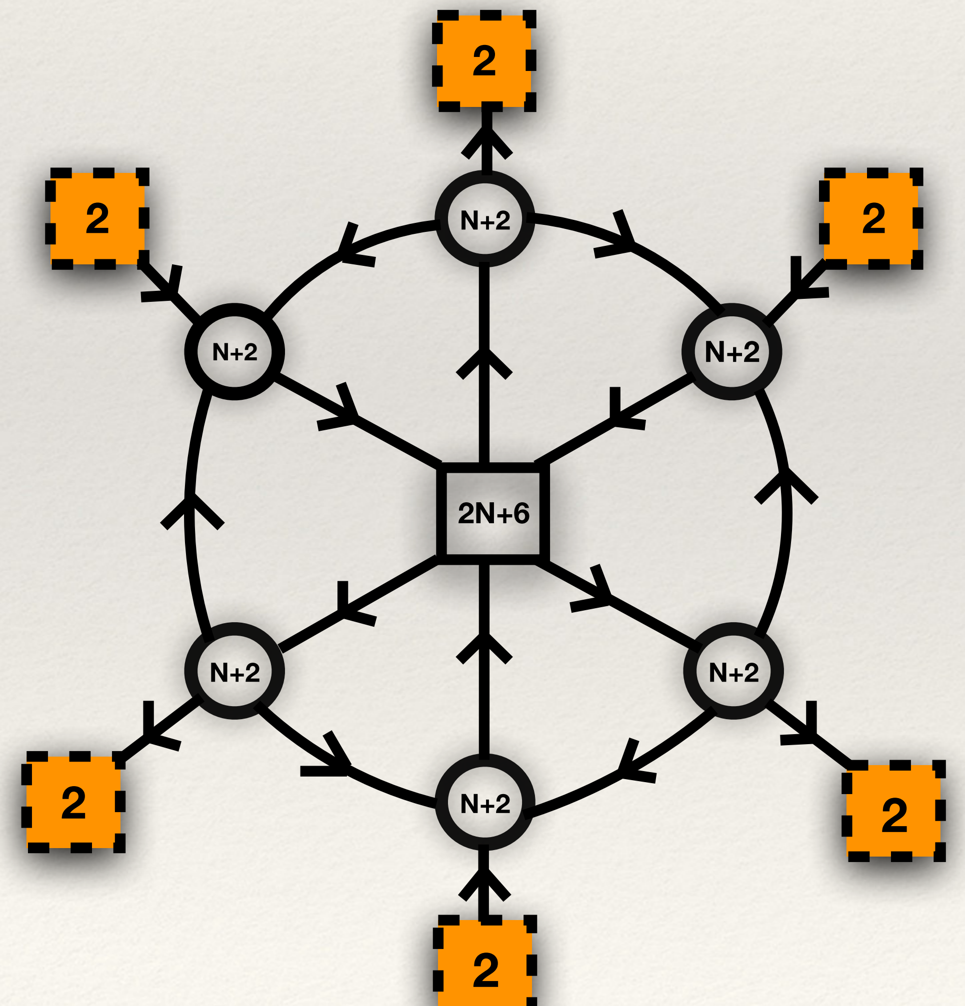
Gluings

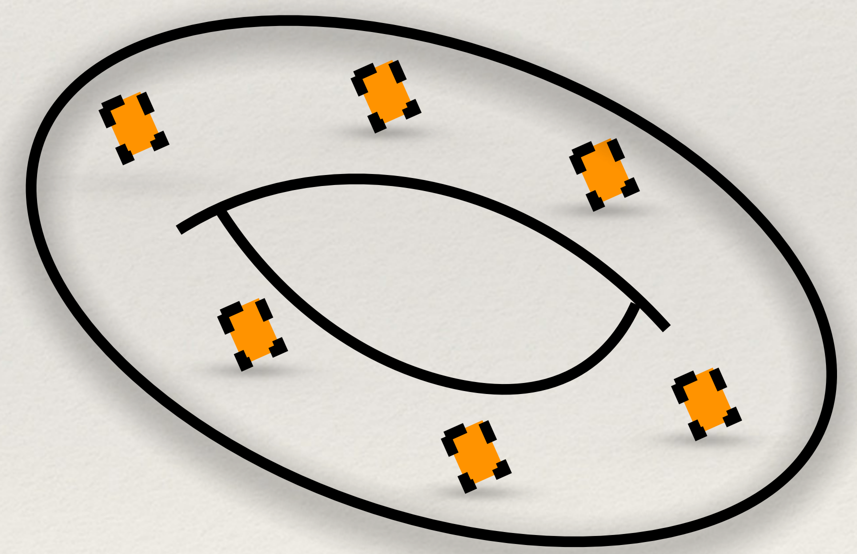
- **S-gluing:**
- Gauge $SU(N+1)$ symmetry and turn on superpotential
- $$W = \sum_{i=1}^{2N+6} M_i M_i'$$
- Flux subtracted

- **Φ-gluing:**
- Gauge $SU(N+1)$ symmetry
- Introduce chiral fundamental fields Φ_i and turn on superpotential
- $$W = \sum_{i=1}^{2N+6} \left(M_i \Phi_i - M_i' \Phi_i\right)$$
- Flux added

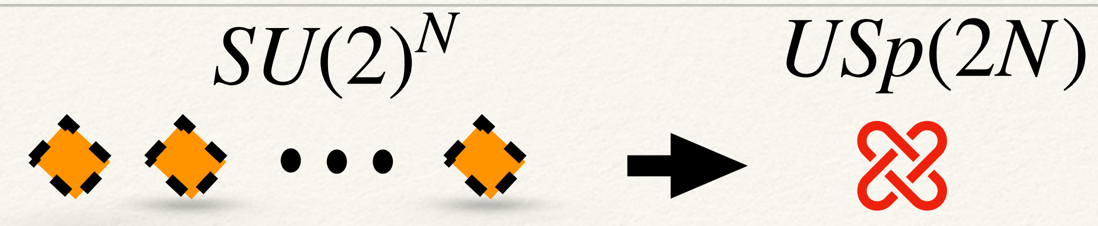
S-gluing into the "Wheel of the Law"

- For example S-gluing $2n$ trinions get a torus with $2n$ minimal punctures
- Use Seiberg Duality





A-typical degenerations:

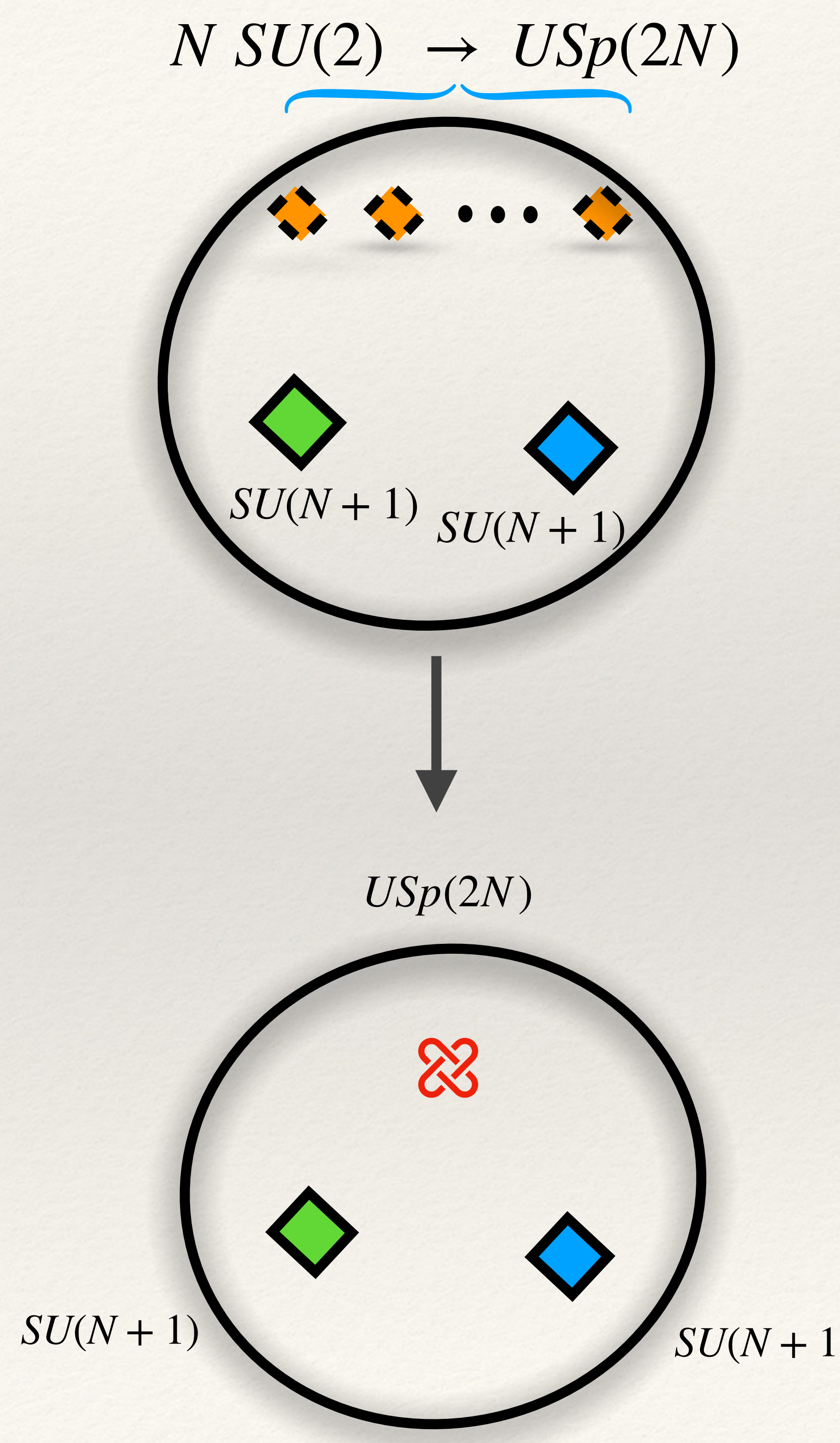


e.g. Chacaltana, Distler, Tachikawa 2012; SR, Vafa, Zafrir 2016; SR, Zafir 2018

- There are three different 5d effective theories:
- $\mathcal{G} = SU(N+1)\,,\ \ Usp(2N)\,,\ \ SU(2)^N$
- Thus there are (at least) **three types of maximal punctures**

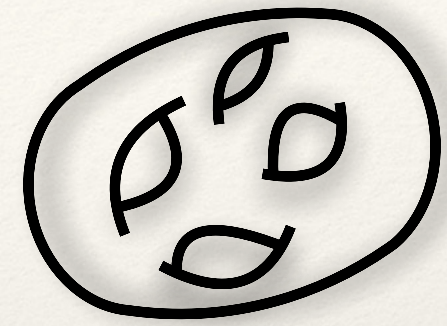
- Claim: N <u>minimal</u> $SU(2)$ **punctures** on the conformal manifold can combine into a <u>maximal</u> $USp(2N)$ **puncture** SR, Sabag 2019

- We can construct a sphere with N minimal $SU(2)$ punctures and two maximal $SU(N+1)$ punctures which on some locus will become 3 maximal
- Can glue the spheres with three maximal punctures into closed Riemann surfaces



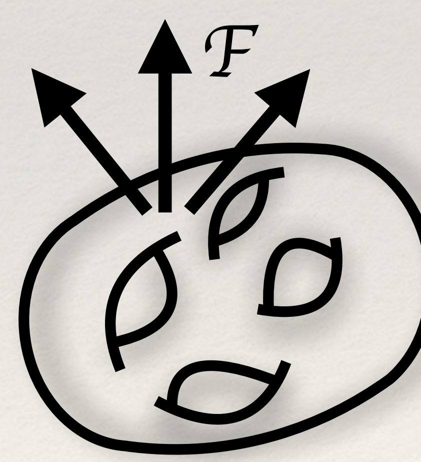
Φ-gluing and the Flux

- S-glue: $\mathscr{G}_F = SO(4N+12)$
- $a = \frac{3}{16}N(16N+9)(g-1)\,,\; c = \frac{1}{8}N(25N+18)(g-1)$
- Φ-glue: $\mathscr{G}_F = SO(4N+8)\times SU(2)\times U(1)$



$$a_{g=2} = \frac{4N\left(10N^2+22N+13\right)^{3/2} + 9\left(16N^3+53N^2+56N+16\right)N}{48(N+2)^2}\,,$$

$\mathscr{F} = (-1,-1,0,0,\cdots)$



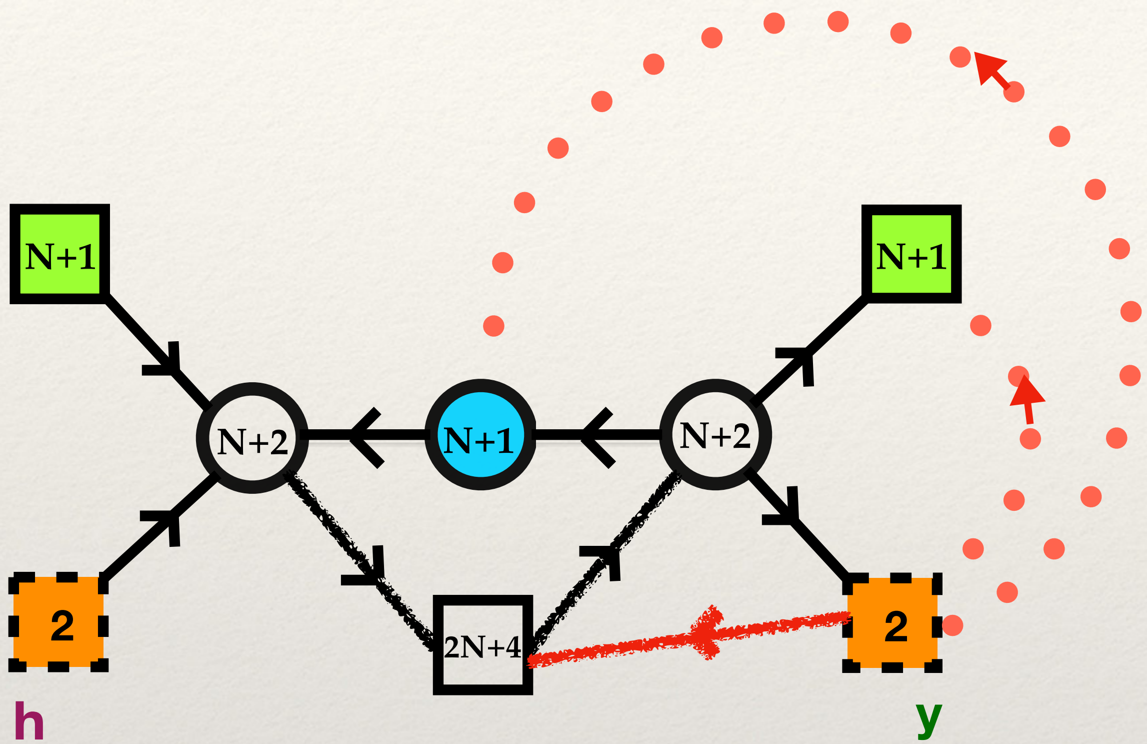
$$c_{g=2} = \frac{2\left(11N^2+26N+17\right)\sqrt{N^2\left(10N^2+22N+13\right)} + 3\left(25N^3+86N^2+98N+34\right)N}{24(N+2)^2}$$

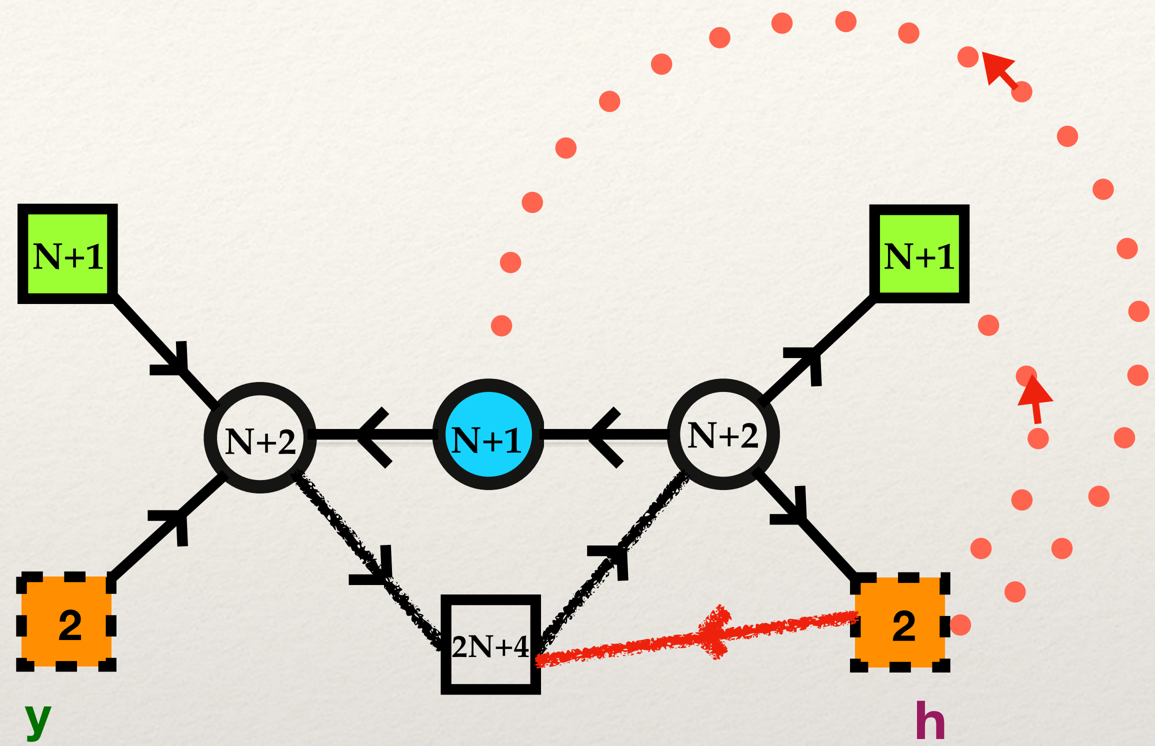
Cartan with the flux

$$SO(4N+12) \;\to\; SO(4N+8)\times SU(2)_{\left(\frac{v}{u}\right)^{(N+1)(N+2)}}\times SU(2)_{\left((vu)^{N+1}w^2\right)^{N+2}}$$

$$\to SU(2N+4)\times U(1)_{\left(\frac{w^2}{uv}\right)^{N+1}}\times SU(2)_{\left(\frac{v}{u}\right)^{(N+1)(N+2)}}\times SU(2)_{\left((vu)^{N+1}w^2\right)^{N+2}}$$

Intriligator-Pouliot duality generalized

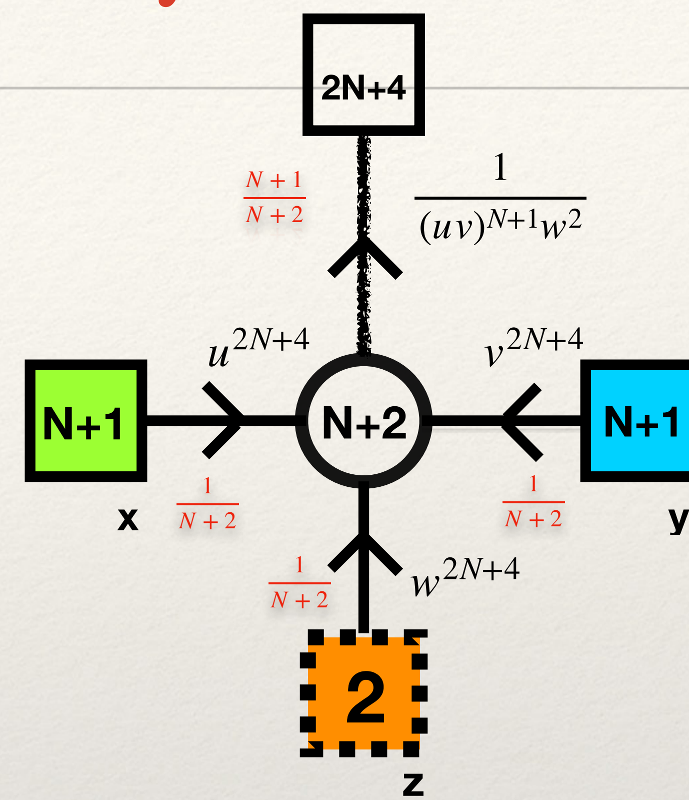


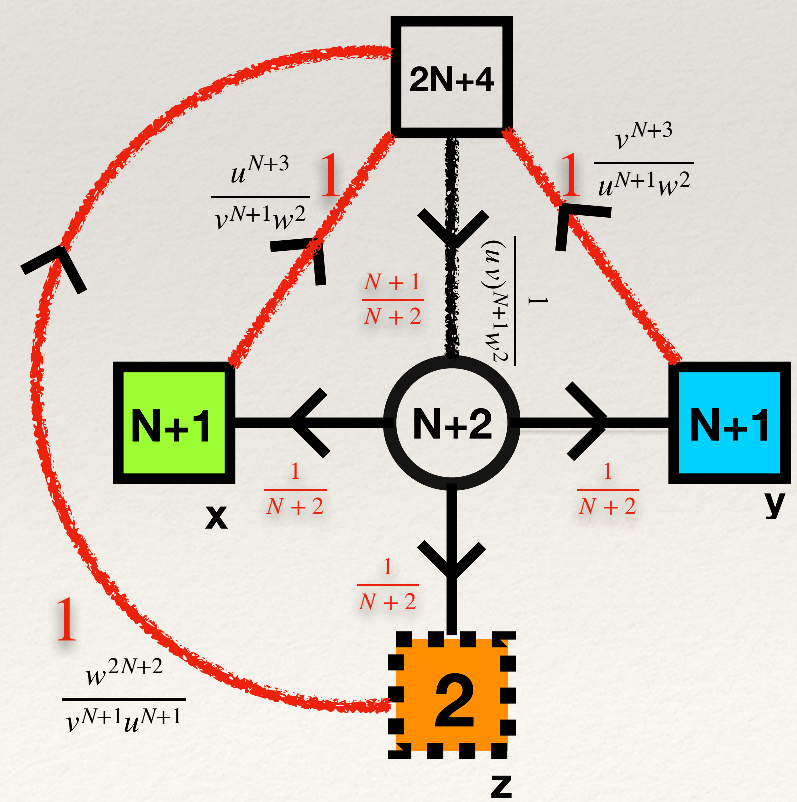


- S-gluing pair of pants decomposition duality
- Take the degenerate $N = 0$ case, gluing is $SU(1)$
- The exchange of puncture symmetry follows from IP duality
- $SU(2)$ $N_f = 4$, flip Mesons and Baryons
- $N > 0$ then a higher rank generalisation of IP duality

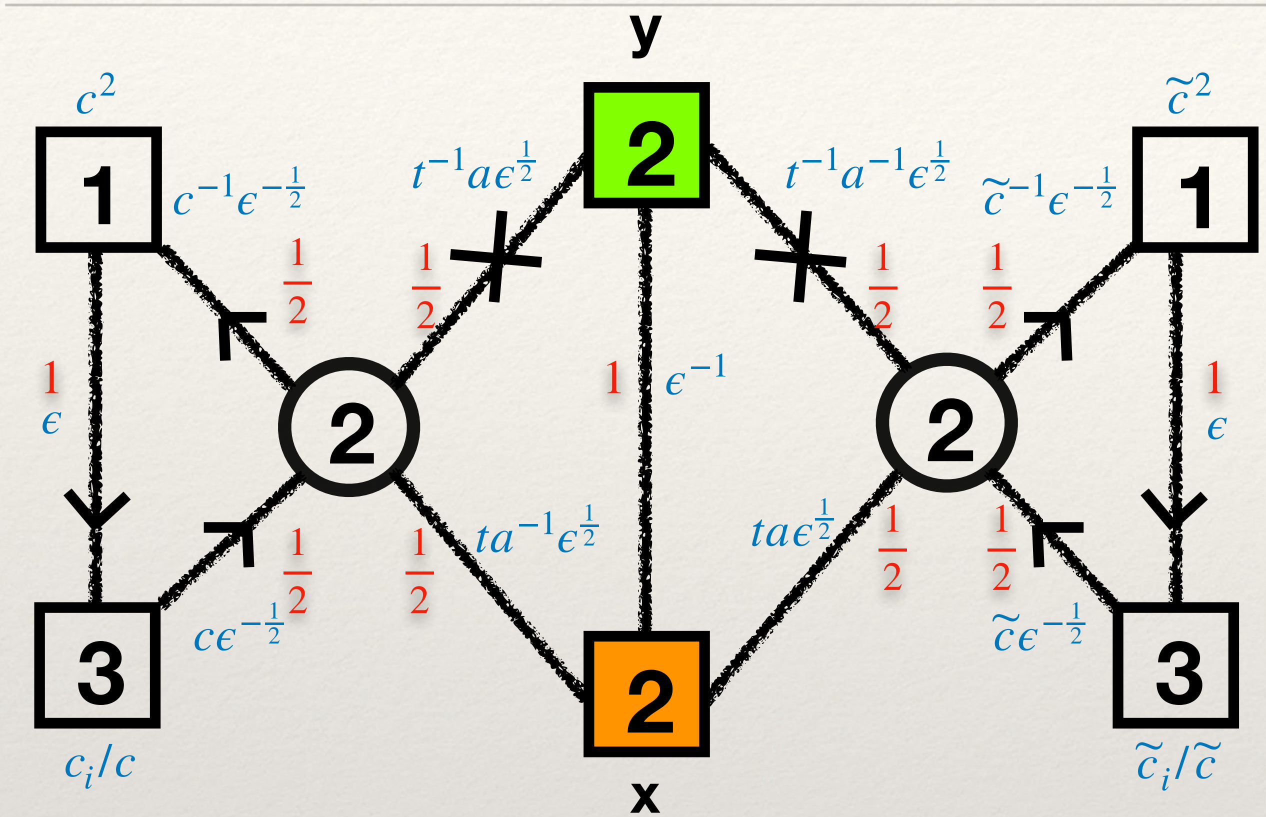
Seiberg duality and geometry

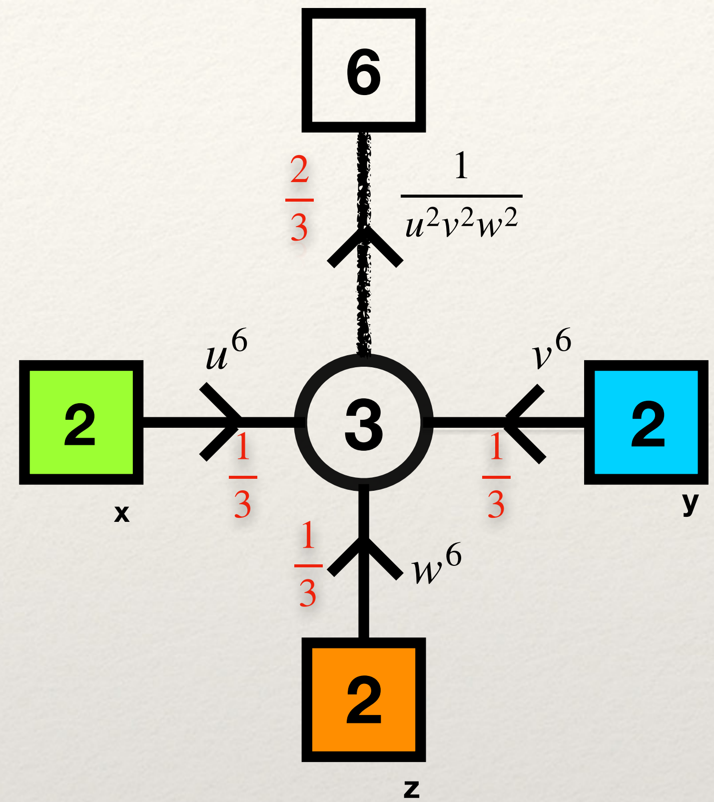
- Seiberg Duality in two steps
- First conjugate $SU(2N+4)^2 \times SU(N+2)_g$
- $\mathcal{F} = (-1, -1, 0, 0, \cdots) \ \rightarrow \ \mathcal{F} = (-1, -1, 0, 0, \cdots)$
- Mesonic moment maps are conjugated
- Second flip the mesons
- The moment maps are the same as originally
- Thus Seiberg duality can be thought of as acting on the theory with an element of the Weyl group of $SO(4N+12)$ conjugating the mesonic $2N+4$ symmetries and exchanging the baryonic symmetries. Flux is invariant, punctures invariant and thus the theories are the same.





4d dualities from 5d dualities

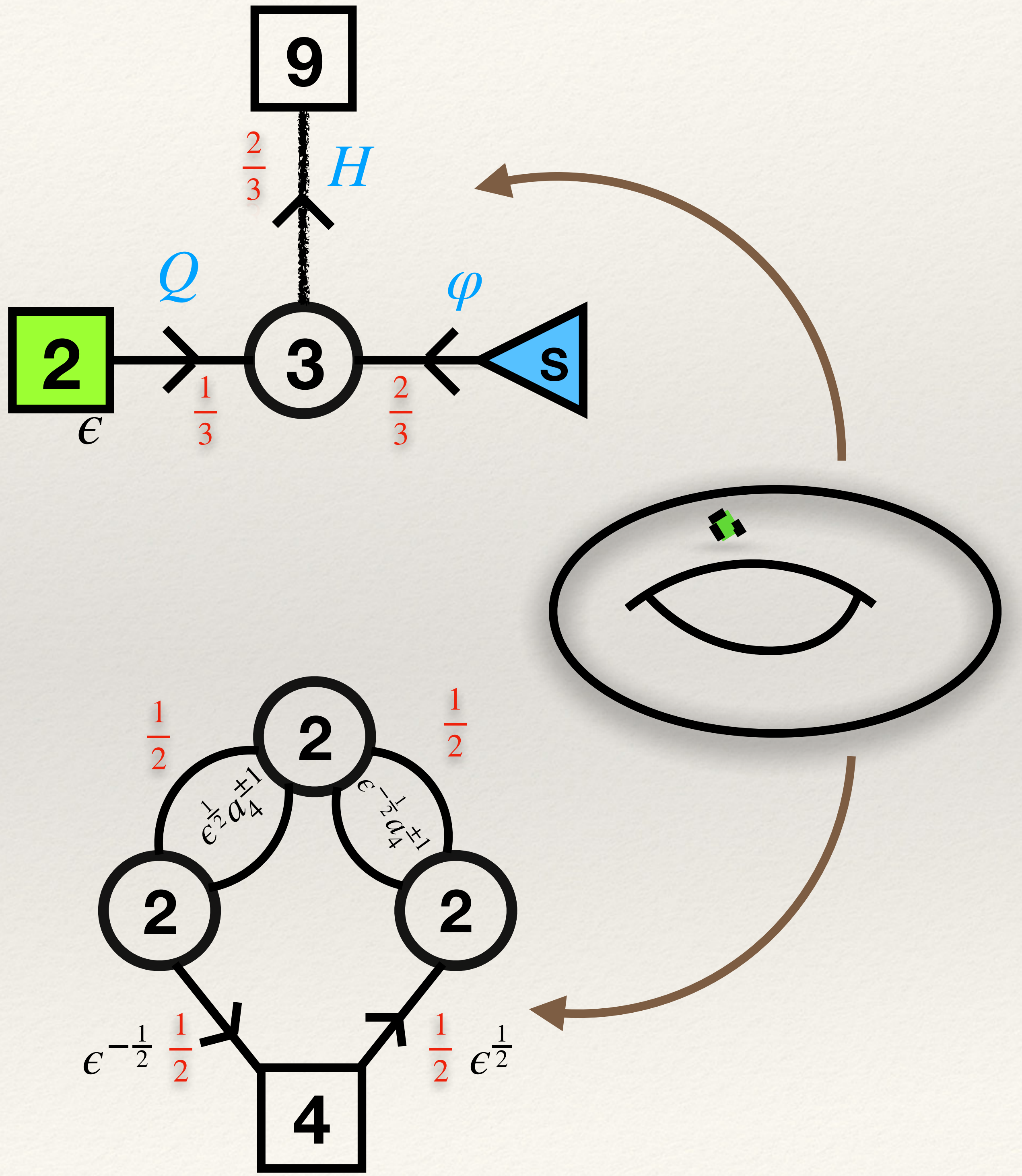




- In fact can construct, in principle, trinions systematically
- Tubes with flux of D_{N+4} flow to trinions of D_{N+3} SR, Sabag, Zafrir 2019; SR, Sabag 2019
- 5d duality between $\mathcal{G} = SU(N+1)\,,\ \ Usp(2N)\,,\ \ SU(2)^N$ leads to different trinions
- On the left trinion using $SU(2)^N$ description ($\mathcal{F} = 0$, $U(1)_\epsilon \to SU(2)_\epsilon$)

Torus with one puncture for the E-string

- Construct torus with one puncture using the two types of trinions
- Glue two punctures of a single trinion
- Different types of punctures, thus break some symmetry
- With suitable superpotentials both constructions preserve same $SO(9)$ and have zero flux
- This is a 4d duality following from 5d duality



Summary and Comments

- $SU(N)$ SQCD in the middle of the conformal window is minimal D conformal matter trinion (two maximal and one minimal punctures)
- Away from the middle, more generic punctures
- Dualities related to Weyl group of $SO(4N+12)$, pairs of pants, and 5d dualities
- Many known dualities and novel ones
- Many cases of emergent symmetries
- Non-trivial dynamics: dangerously irrelevant operators
- A-typical degenerations: Gauging emergent symmetries
- Simple and non-Simple groups and dualities treated uniformly
- All 5d effective description are important
- *(Often 6d reductions give complicated 4d theories (see class $\mathcal{S}$), here get simple 4d theories)*

Outlook

- Is everything Lagrangian?
- The space of flows starting from free SCFTs and allowing dangerously irrelevant deformations and gauging emergent symmetries covers all SCFTs in 4d?
- Do all non-trivial 4d phenomena have a simple 6d geometric interpretation?
- Do all models with four supercharges come from SCFTs with eight supercharges?