Higgs Branches, Magnetic Quivers, and Hasse Diagrams

Marcus Sperling

Yau Mathematical Sciences Center, Tsinghua University (Beijing)

Strings and QFTs for Eurasian time zone

— July 8, 2020 —

Motivation

Higgs branch $\mathcal{H}$ of

d=$3,\ldots,6$ theory w/ 8 SUSY

- classical exact
- $\mathcal{H}|_{g^2<\infty}=\{\text{F-terms}=0\}/G^{\mathbb{C}}$
- hyper-Kähler quotient

Coulomb branch $\mathcal{C}$ of

d=3 theory w/ 8 SUSY

- quantum corrections
- $\mathcal{C}=\{\text{dressed monopole op.}\}$
- symplectic singularity

Today's question: $\mathcal{H}|_{g^2=\infty} = ???$

Known changes of $\mathcal{H}$

- 6d: tensionless strings – e.g. small E_8 instanton [Hanany, Ganor '96]
- 5d: instanton operators – e.g. $\mathrm{SU}(2)$ w/ N_f [Seiberg '96; Morrison, Seiberg '97]

$$\mathcal{H}_{g^2<\infty} = \overline{\mathcal{O}}^{\min}_{D_{N_f}} \to \mathcal{H}_{g^2=\infty} = \overline{\mathcal{O}}^{\min}_{E_{N_f+1}}$$

Proposed answer to: $\mathcal{H}|_{g^2=\infty} = ???$

① Derive magnetic quiver such that

$$\mathcal{H}|_{g^2=\infty} = \mathcal{C}\,(\text{magnetic quiver})$$

as **equality of moduli spaces**

[2004.04082: Bourget, Grimminger, Hanany, MS, Zhong]

[1912.02773: Cabrera, Hanany, MS]

[1904.12293: Cabrera, Hanany, MS]

② Derive **Hasse diagram** for $\mathcal{H}|_{g^2=\infty}$ such that

- $\mathcal{H}|_{g^2=\infty}$ foliated into symplectic leaves $\mathcal{L}_\kappa$
- with transverse slices $\mathcal{S}_{\kappa,\lambda}$ for $\mathcal{L}_\kappa < \mathcal{L}_\lambda$

for **each slice** and **each leaf**: $\exists$ magnetic quiver

[1908.04245: Bourget, Cabrera, Grimminger, Hanany, MS, Zajac, Zhong]

Outline

1. Magnetic quivers from 5-brane webs
   - 5-brane webs with O7 planes
   - 5-brane webs with O5 planes
2. Hasse diagrams
   - Finite Coupling
   - Infinite Coupling
3. Exceptional families
4. Conclusions and outlook

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5-brane webs in Type IIB

- Simultaneously: 5d gauge theory dynamics and UV fixed points
- **Enhancement of global symmetry** due to instanton operators

$$\text{topological } \mathrm{U}(1)_I : \qquad \mathrm{tr} \star (F \wedge F)$$

- **Enhancement of $\mathcal{H}$** at fixed point?
- First studies of $\mathcal{H}_\infty$ for $\mathrm{SU}(k)$ SQCD with fundamentals

  [Cremonesi, Ferlito, Hanany, Mekareeya '15; Ferlito, Hanany, Mekareeya, Zafrir '17]

  $\longrightarrow$ Magnetic quiver [Cabrera, Hanany, Yagi '18]
- 5d $\mathrm{Sp}(k)$ or $\mathrm{SO}(k)$ SQCD with fundamental flavours
  - field theory [Intrilligator, Morrison, Seiberg '97]
  - 5-brane webs + orientifolds [Brunner, Karch '97; Bergman, Zafrir '15; Zafri '16]

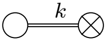
Aim: Improve with magnetic quivers

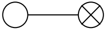
Rules for brane configurations

Type II brane configurations NS5–Dp–D$(p+2)$ [Hanany, Witten '96]

with NS5 $\otimes$ Dp ——— D$(p+2)$ $\bigcirc$

Rules:

- 

only if $k = 0, 1$: SUSY condition / S-rule
-  $\leftrightarrow$ $\otimes$ $\bigcirc$ brane creation / annihilation
- $+$ rules for orientifolds O$p^{\pm}$, $\widetilde{\mathrm{O}p}^{\pm}$

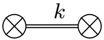
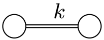
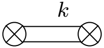
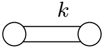
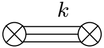
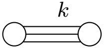
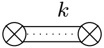
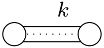
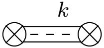
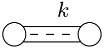
Electric phase: Dp suspended between NS5

$\longrightarrow$ low energy effective (electric) theory

Higgs branch phase: Dp suspended between D$(p+2)$

$\longrightarrow$ magnetic quiver?

Magnetic quivers

conventional **electric theory** Dp between NS5s			proposed **magnetic theory** Dp between D$(p+2)$		
k Dp		$\mathrm{SU}(k)$	k Dp		U_k
5d: [Cabrera, Hanany, Yagi '18], 6d: [Cabrera, Hanany, MS '19]					
k Dp Op^-		$\mathrm{SO}(2k)$	k Dp Op^-		D$_k$
k Dp $\widetilde{\text{O}p}^-$		$\mathrm{SO}(2k{+}1)$	k Dp Op^+		C$_k$
k Dp Op^+		$\mathrm{USp}(2k)$	k Dp $\widetilde{\text{O}p}^-$		B$_k$
k Dp $\widetilde{\text{O}p}^+$			k Dp $\widetilde{\text{O}p}^+$		C$_k$
5d: [Bourget, Grimminger, Hanany, MS, Zhong '20], 6d: [Cabrera, Hanany, MS '19]					

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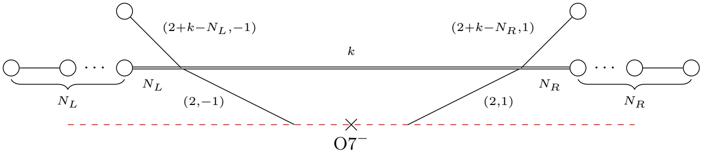
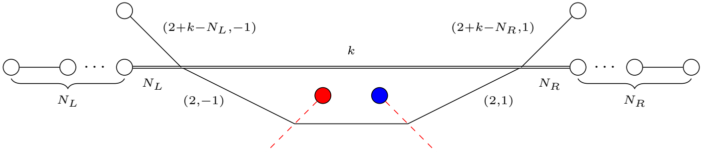
5-brane webs with O7$^-$ planes

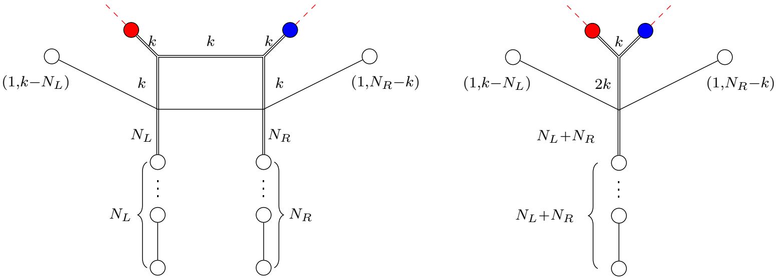
5-brane web for $\mathrm{Sp}(k)$ with $N_f = N_L + N_R$ fundamental flavours

constraint: $N_f \leqslant 2k+5$ [Bergman, Zafrir '15]



Quantum: O7$^-$ resolved to $[1,1]$ 7-brane + $[1,-1]$ 7-brane [Sen'96]

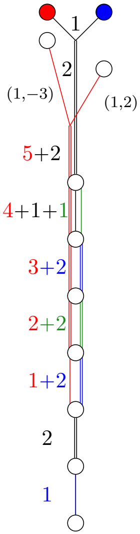




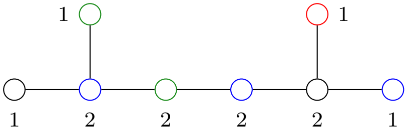
5-brane web suspended on 7-branes

- unitary magnetic quiver via [Cabrera, Hanany, Yagi '18]

E_8 example: finite coupling

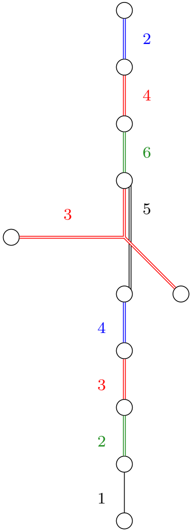


- electric theory: $\mathrm{Sp}(1)$ with $N_f\!=\!7$
- magnetic gauge nodes
  $\longleftrightarrow$ maximal subdivision
- magnetic hypermultiplets
  $\longleftrightarrow$ intersection number

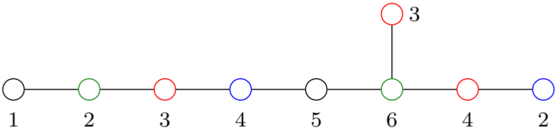


$$\mathcal{C} = \overline{\mathcal{O}}^{\mathrm{min}}_{D_7} = \mathcal{H}_{g<\infty}\big(\mathrm{Sp}(1),N_f\!=\!7\big)$$

E_8 example: infinite coupling



- magnetic gauge nodes
  $\longleftrightarrow$ maximal subdivision
- magnetic hypermultiplets
  $\longleftrightarrow$ intersection number



$$\mathcal{C} = \overline{\mathcal{O}}^{\min}_{E_8} = \mathcal{H}_\infty\big(\mathrm{Sp}(1), N_f{=}7\big)$$

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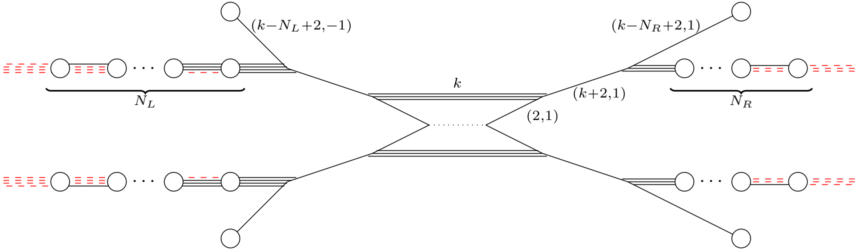
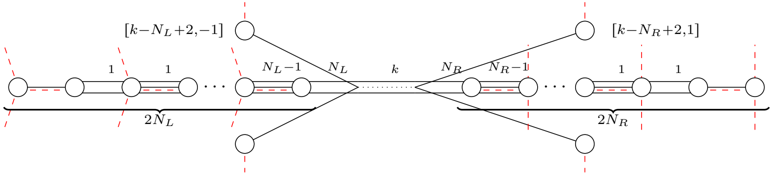
5-branes with O5$^-$ planes

5-brane web for $\mathrm{Sp}(k)$ with $N_f = N_L + N_R$ fundamental flavours [Zafrir '15]

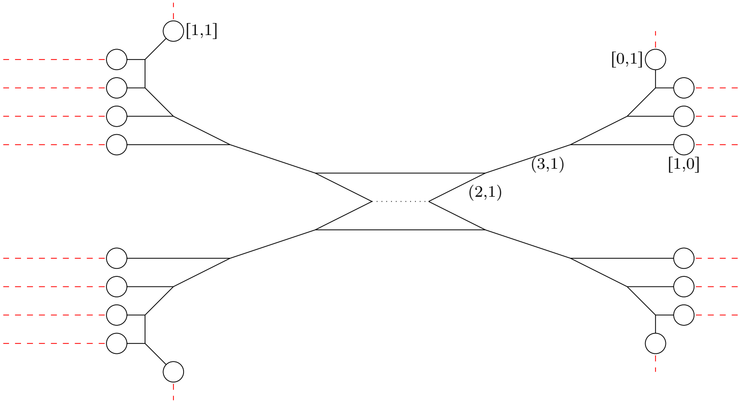


Higgs branch phase

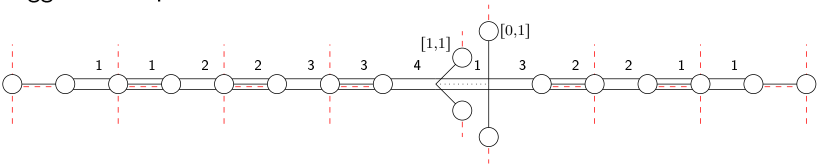


E_8 example: finite coupling

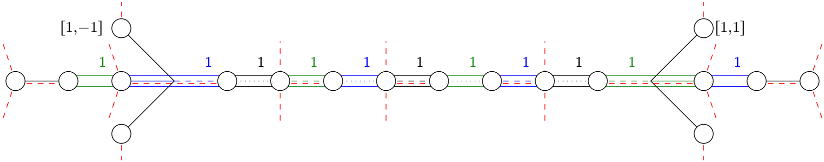
5-brane web for $\mathrm{Sp}(1)$ with 7 fundamental flavours



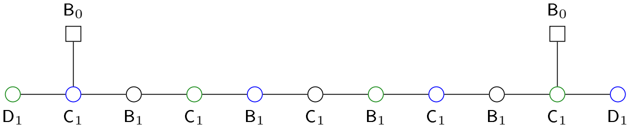
Higgs branch phase



Annihilate frozen branes



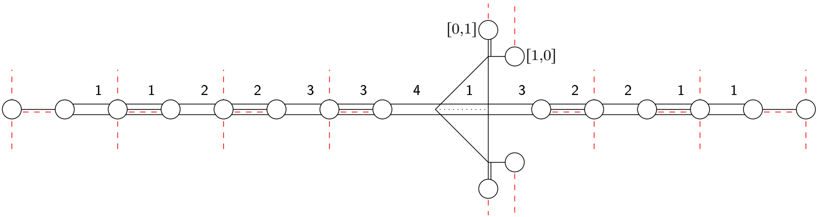
Magnetic quiver (via max subdivision and intersection number)



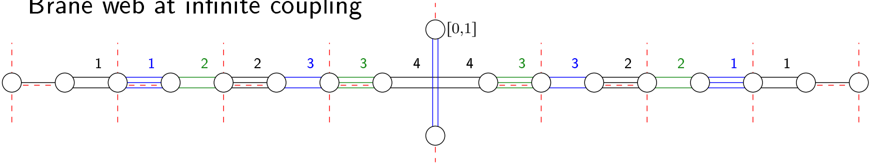
$$\mathcal{C} = \overline{\mathcal{O}}^{\text{min}}_{D_7} = \mathcal{H}_{g<\infty}\big(\text{Sp}(1), N_f = 7\big)$$

[Benini, Tachikawa, Xie '10]
[Chacaltana, Distler, Tachikawa '13]
[Cabrera, Hanany, Zhong '17]

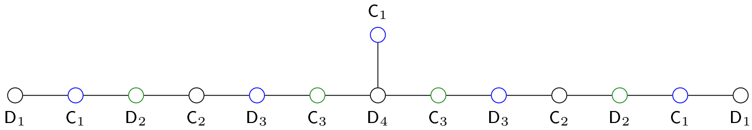
E_8 example: infinite coupling



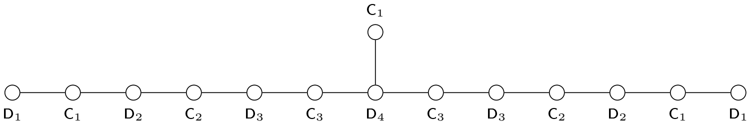
Brane web at infinite coupling



Magnetic quiver (via max subdivision and intersection number)



Subtlety: $\mathcal{C}$ (unframed ortho-symplectic magnetic quiver) $= ???$



- Same **magnetic quiver** for E-string: M5 on D_4 singularity

  [Hanany, Mekareeya '18; Cabrera, Hanany, MS '19]

- **Class** $\mathcal{S}$: sphere with 2 max and 1 min puncture of $\mathrm{SO}(8)$

  $\longrightarrow E_8$ SCFT [Chacaltana, Distler '11; Ohmori, Shimizu, Tachikawa, Yonekura '15]

- **Hilbert series** validation [Zhong, MSc Thesis '18]

  [Bourget, Grimminger, Hanany, Kalveks, MS, Zhong – to appear soon]

Conclusion:

$$\mathcal{C} = \overline{\mathcal{O}}_{E_8}^{\mathrm{min}} = \mathcal{H}_\infty \left(\mathrm{Sp}(1), N_f{=}7\right)$$

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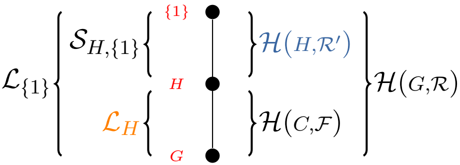
Higgs branch Hasse diagram

Example: 8 SUSY, Lagrangian

G gauge theory with matter content $\mathcal{R}$ $\rightarrow$ Higgs branch $\mathcal{H}(G,\mathcal{R})$



- $H \subset G$ s.t. $\mathcal{R}|_H = \oplus_i a_i \mathbf{r}_i$ and $\mathrm{Adj}^G|_H = \mathrm{Adj}^H + \oplus_i b_i \mathbf{r}_i$
- Higgs $G \xrightarrow{\mathrm{VEV}} H$: $a_i \geqslant b_i$, $\forall i$
- H **gauge theory** with matter $\mathcal{R}' = \oplus_{i \neq \mathrm{triv}} (a_i - b_i)\mathbf{r}_i$
- $(a_{\mathrm{triv}} - b_{\mathrm{triv}})$ **singlets**
- $C = C_G(H)$, $\dim(C) = b_{\mathrm{triv}}$ w/ a_{triv} hypers in $\mathcal{F}$ of C

$\mathcal{L}_H$, $\mathcal{L}_{\{1\}}$ ... symplectic leaves

$\mathcal{S}_{H,\{1\}}$ ... transverse slice

Hasse diagram

Symplectic singularties [Beauville '00]

admit foliation/stratification $\{\mathcal{L}_\kappa\}$ into symplectic leaves

- mutally disjoint $\mathcal{L}_\kappa \cap \mathcal{L}_\lambda = \emptyset$ for $\kappa \neq \lambda$
- partially ordered $\mathcal{L}_\kappa < \mathcal{L}_\lambda \Leftrightarrow \mathcal{L}_\kappa \subset \overline{\mathcal{L}_\lambda} \leftrightarrow$ Hasse diagram
- closure $\overline{\mathcal{L}_\kappa}$ = symplectic singularity
- ordered pair $(\mathcal{L}_\kappa, \mathcal{L}_\lambda)$, $\mathcal{L}_\kappa < \mathcal{L}_\lambda$
  $\rightarrow$ **transverse slice** $\mathcal{S}_{\kappa,\lambda}$ =symplectic singularity

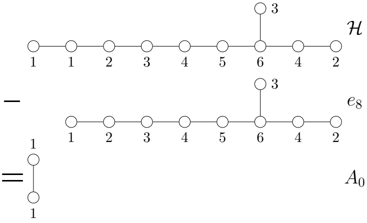
Nilpotent orbit closures:

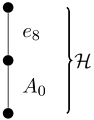
minimal transverse slices $\mathcal{S}$ classified [Kraft, Procesi '80]

- Kleinian singularities A_n, D_n, $E_{6,7,8}$ or
- minimal nilpotent orbit closures a_n, d_n, $e_{6,7,8}$, ...

Quiver subtraction [Cabrera, Hanany '18]

- given magnetic quiver for $\mathcal{H}$
- identify minimal transverse slices
  - → need complete list
  - → extend [Kraft, Procesi '80]
- subtract quiver realisation
  - → need all possible realisations
- read Hasse diagram





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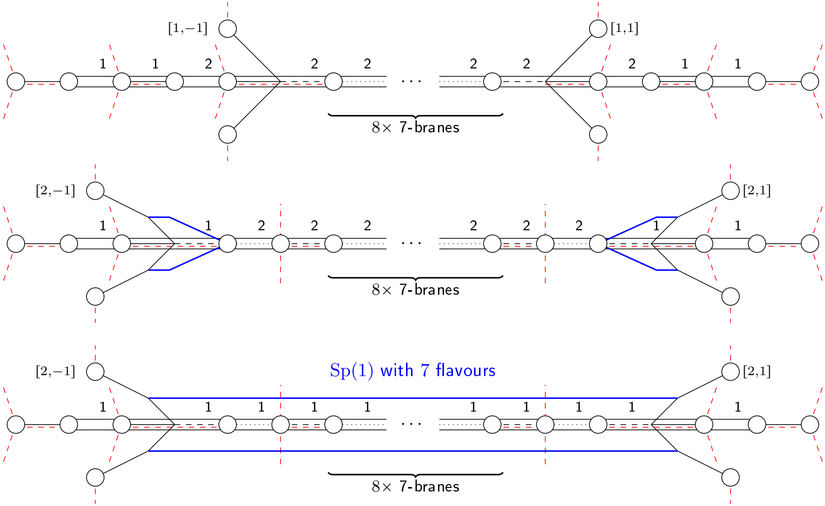
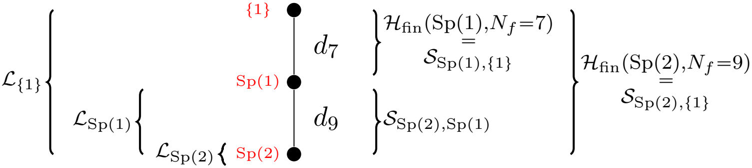
Finite Coupling

Example: $\mathrm{Sp}(2)$ with 9 flavours

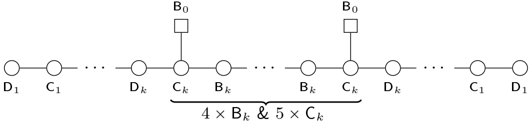


symplectic leaves **transverse slices**



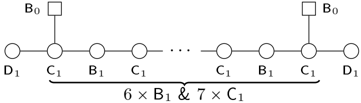
with

$\mathcal{C}\Big($



$\Big) = \begin{cases} \mathcal{S}_{\mathrm{Sp}(2),\{1\}}, & k=2 \\ \mathcal{S}_{\mathrm{Sp}(1),\{1\}}, & k=1 \end{cases}$

$\mathcal{C}\Big($



$\Big) = \mathcal{S}_{\mathrm{Sp}(2),\mathrm{Sp}(1)}$

same result via **quiver subtraction** [Cabrera, Hanany '18]

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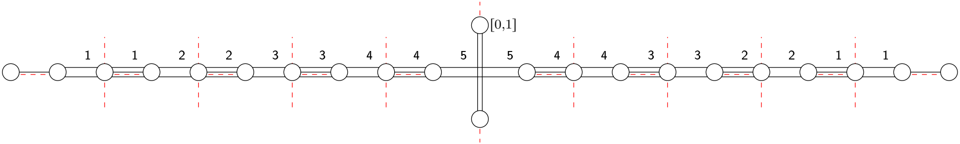
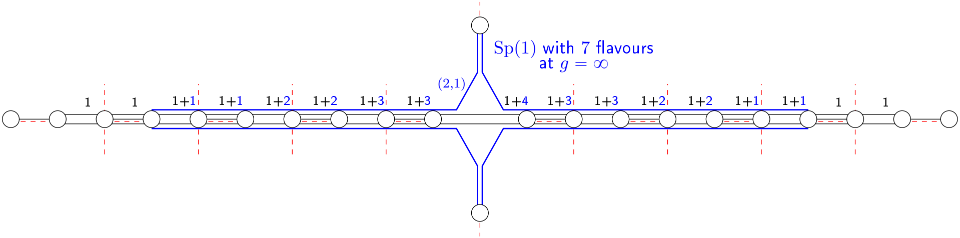
Infinite Coupling

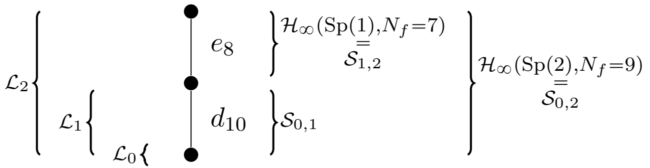
Example: $\mathrm{Sp}(2)$ with 9 flavours



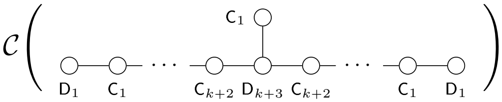
open up 5d Coulomb branch direction



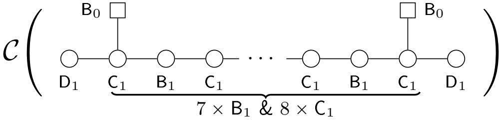
symplectic leaves | transverse slices



with



$$= \begin{cases} \overline{\mathcal{L}}_2 & k = 2 \\ \mathcal{S}_{1,2} & k = 1 \end{cases}$$



$$= \overline{\mathcal{L}}_1$$

same result via **quiver subtraction** [Cabrera, Hanany '18]

Comparison: finite vs infinite coupling

	finite coupling	infinite coupling	symmetry enhancement
$\mathrm{Sp}(1)\ N_f = 7$	d_7	e_8	$\mathrm{SO}(14) \to E_8$
$\mathrm{Sp}(2)\ N_f = 9$	d_7, d_9	e_8, d_{10}	$\mathrm{SO}(18) \to \mathrm{SO}(20)$

- **non-abelian part of global symmetry** via transverse slice to origin
- $\mathrm{U}(1)_I$ not in geometry of $\mathcal{H}_{g<\infty}$ $\leftrightarrow$ **nilpotent gaugino bilinear**
- matches symmetry enhancement [Bergman, Zafrir '15, Zafrir '15]

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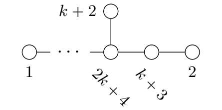
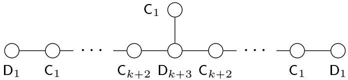
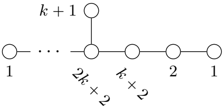
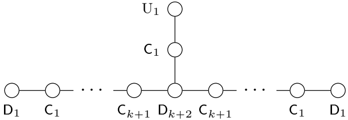
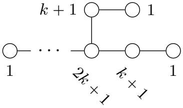
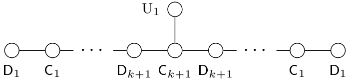
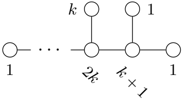
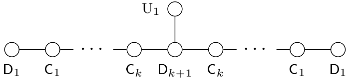
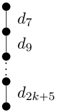
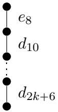
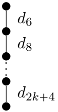
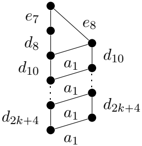
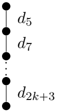
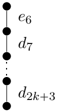
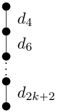
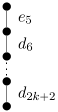
Exceptional Families

Family	Theory SU	Theory Sp	Magnetic quiver U	Magnetic quiver OSp
E_8	$2k{+}5$ — $\mathrm{SU}(k{+}1)_{\pm\frac{1}{2}}$	D_{2k+5} — $\mathrm{Sp}(k)$		
E_7	$2k{+}4$ — $\mathrm{SU}(k{+}1)_{\pm 1}$	D_{2k+4} — $\mathrm{Sp}(k)$		
E_6	$2k{+}3$ — $\mathrm{SU}(k{+}1)_{\pm\frac{3}{2}}$	D_{2k+3} — $\mathrm{Sp}(k)$		
E_5	$2k{+}2$ — $\mathrm{SU}(k{+}1)_{\pm 2}$	D_{2k+2} — $\mathrm{Sp}(k)$		

5d Duality [Gaiotto, H.-C. Kim '15; Hayashi, S.-S. Kim, K. Lee, Yagi '15]

Family	Dimension and symmetry for $k>1$ at finite coupling	Dimension and symmetry for $k>1$ at infinite coupling	Hasse diagram finite coupling	Hasse diagram infinite coupling
E_8	$2k^2+9k$ $\mathfrak{so}(4k+10)$	$2k^2+11k+16$ $\mathfrak{so}(4k+12)$		
E_7	$2k^2+7k$ $\mathfrak{so}(4k+8)$	$2k^2+7k+8$ $\mathfrak{so}(4k+8)\oplus\mathfrak{su}(2)$		
E_6	$2k^2+5k$ $\mathfrak{so}(4k+6)$	$2k^2+5k+4$ $\mathfrak{so}(4k+6)\oplus\mathfrak{u}(1)$		
E_5	$2k^2+3k$ $\mathfrak{so}(4k+4)$	$2k^2+3k+2$ $\mathfrak{so}(4k+4)\oplus\mathfrak{u}(1)$		

matches symmetry enhancement [Bergman, Zafrir '15; Zafrir '15]

Outline

1. Magnetic quivers from 5-brane webs
   - 5-brane webs with O7 planes
   - 5-brane webs with O5 planes
2. Hasse diagrams
   - Finite Coupling
   - Infinite Coupling
3. Exceptional families
4. Conclusions and outlook

Status on Magnetic Quivers

Magnetic quiver

- Applicable for finite and infinite coupling Higgs branches
- Predictions for moduli spaces

Type II brane constructions

- without Op: unitary magnetic quivers – many advantages
- with Op: orthosymplectic magnetic quivers – more subtle

Not limited to brane constructions

- rank 1 4d $\mathcal{N}=2$ SCFTs [Bourget, Grimminger, Hanany, MS, Zafrir, Zhong '20]

Status on Hasse Diagrams

Derivable from

- Partial Higgsing for Lagrangian theories at finite coupling
- **KP transitions** in brane configurations
- **Quiver subtraction** on magnetic quivers

Necessary data

- Complete list of minimal degenerations – extend [Kraft, Processi '80]
  $\longrightarrow$ new proposed slices [Bourget, Grimminger, Hanany, MS, Zafir, Zhong '20]
- Realisations as unitary and orthosymplectic magnetic quivers
  $\longrightarrow$ e.g. exceptional families