Surface Defects from Fractional Branes

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Based on work done with . . .

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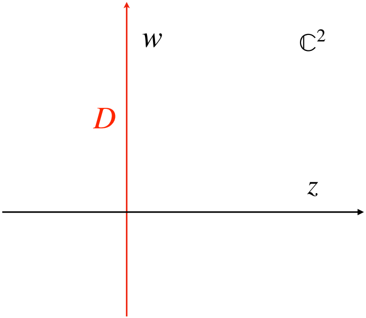
Outline

- Review of surface defects in $\mathcal{N} = 4$ Yang-Mills theory.
  Gukov, Witten, '06
- Set-up: fractional D3 branes on orbifolds.
  Kanno-Tachikawa, '10
- Overview of main idea.
- Open/closed disk amplitudes on the worldsheet:
  - Twisted closed string spectrum on $\mathbb{C}^2/\mathbb{Z}_2$.
  - Fractional D3 branes: reflection rules + open strings
- Gauge theory profiles from disk amplitudes.

Review of surface defects

- Surface defects are co-dimension 2 defects in gauge theories. Higher dimensional generalizations of Wilson and ’t Hooft line defects.
- We focus on such defects in $\mathcal{N} = 4$ SYM in four dimensions with gauge group U(N).
- The defects we study will be 2-planes $\mathbb{R}^2 \subset \mathbb{R}^4$.
- These were introduced by Gukov and Witten as monodromy defects.



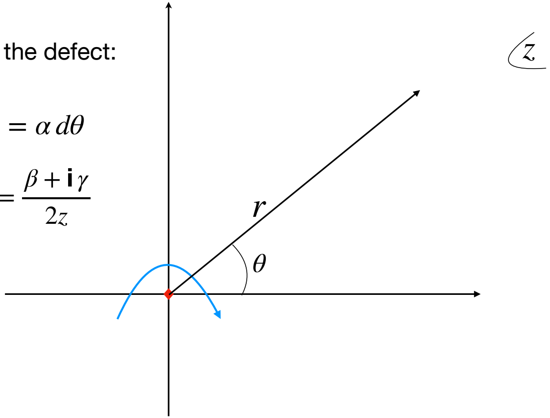
Review of surface defects

- The holonomy of the gauge field around the location of the defect is non-zero.

Near the defect:

$$\mathbf{A} = \alpha \, d\theta$$

$$\Phi = \frac{\beta + \mathbf{i}\gamma}{2z}$$



Review of surface defects

For a generic surface defect, the profiles of the gauge field and scalar take the following form: Gukov, Witten, '06

$$\mathbf{A} = \begin{pmatrix} \alpha_0\,\mathbb{I}_{n_0} & 0 & \cdots & 0 \\ 0 & \alpha_1\,\mathbb{I}_{n_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_{M-1}\,\mathbb{I}_{n_{M-1}} \end{pmatrix} d\theta\ ,$$

and

$$\Phi = \begin{pmatrix} (\beta_0 + \mathrm{i}\,\gamma_0)\,\mathbb{I}_{n_0} & 0 & \cdots & 0 \\ 0 & (\beta_1 + \mathrm{i}\,\gamma_1)\,\mathbb{I}_{n_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (\beta_{M-1} + \mathrm{i}\,\gamma_{M-1})\,\mathbb{I}_{n_{M-1}} \end{pmatrix} \frac{1}{2z}\ .$$

The field configuration breaks the U(N) gauge group to a subgroup at the location of the defect:

$$\mathrm{U}(n_0) \times \mathrm{U}(n_1) \times \cdots \times \mathrm{U}(n_{M-1})\ .$$

Review of surface defects

- The integers n_I are a partition of N:

$$\sum_{I=0}^{M-1} n_I = N \, .$$

- The parameters $(\alpha_I, \beta_I, \gamma_I)$ specify the singularity of the gauge field and scalar at the location of the defect.

Review of surface defects

- In the path-integral one is allowed to turn on a 2*d* θ-term, whose coefficient we denote η_I for each factor in the unbroken Levi subgroup:

$$\exp\left(\mathrm{i}\sum_{I=0}^{M-1}\eta_I\int_D \mathrm{Tr}_{\,\mathrm{U}(n_I)}F_I\right)\ .$$

- The Gukov-Witten defect is therefore characterized by the discrete parameters n_I, which constitute a partition of N, and by the four sets of real continuous parameters

$$\{\alpha_I,\beta_I,\gamma_I,\eta_I\}\ ,\qquad \text{with}\quad I=0,\ldots,M-1\ .$$

Review of surface defects: S-duality

- $\mathcal{N} = 4$ Yang-Mills theory is invariant under the action of the non-perturbative duality group SL$(2, \mathbb{Z})$.

- It turns out that this duality group also acts naturally on the parameters of the surface defect. In particular, an element $\Lambda = \left(\begin{smallmatrix} m & n \\ p & q \end{smallmatrix}\right) \in$ SL$(2, \mathbb{Z})$ induces the transformation

$$\begin{aligned} (\alpha_l, \eta_l) &\longrightarrow (\alpha_l, \eta_l)\,\Lambda^{-1} = (q\,\alpha_l - p\,\eta_l, -n\,\alpha_l + m\,\eta_l)\,, \\ (\beta_l, \gamma_l) &\longrightarrow |p\,\tau + q|\,(\beta_l, \gamma_l) \end{aligned}$$

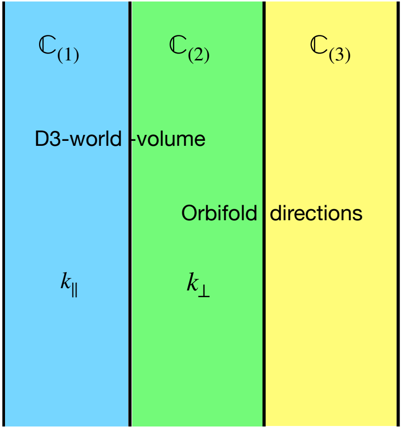
  where τ is the complexified gauge coupling constant.

Goal

- Our goal is to embed or realize the Gukov-Witten defect in string theory.
- This means a derivation of the profiles of the gauge field and scalar using worldsheet computations.
- We will do so by engineering the gauge theory + defect using fractional D3-branes on an orbifold background of type IIB string theory.
- We will verify our proposal by checking consistency with the S-duality properties of the parameters.

Set-up: defects from orbifolds

- The basic idea goes back to Kanno and Tachikawa (KT) in 2010, who considered n_I stacks of fractional D3-branes on the orbifold:
$$\mathbb{C}_{(1)} \times \frac{\mathbb{C}_{(2)} \times \mathbb{C}_{(3)}}{\mathbb{Z}_M} \times \mathbb{C}_{(4)} \times \mathbb{C}_{(5)} \,.$$
- The D3-branes extend along $\mathbb{C}_{(1)}$ and $\mathbb{C}_{(2)}$ (partially extended along the orbifold directions).



Evidence for the proposal

- Kanno-Tachikawa calculated the instanton partition function in the presence of a surface defect.

- They considered stacks of fractional D3 and D(-1) branes in this background and calculated the effective action of open strings with at least one end point on the D(-1) branes (the moduli action). The resulting theory was an orbifold of the usual ADHM moduli action.

- For $\mathcal{N} = 2$ examples, the low energy effective action of the defect on the Coulomb branch of the 4d theory can then be matched with those obtained from other approaches.

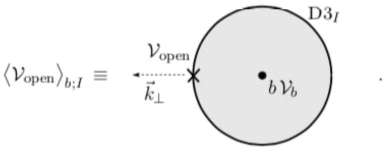
- See also work by Jeong-Nekrasov.

Basic idea of our approach

- We take this D3-branes-on-orbifold picture seriously and ask: is it possible to recover the profiles of the gauge field and scalar?
- The profiles imply that there should be a non-vanishing source (one-point function) for the open string excitations.
- Since the defect is realized by the orbifold, it must be due to states special to the orbifold: twisted sector states.
- We turn on constant background values for twisted sector scalars and check if this induces a source for the open string excitations.

From worldsheet correlators to profiles

- We study open strings on $\sum_{I=1}^{M} n_I = N$ fractional D3-branes and especially, how they couple to the closed string scalars in the twisted sectors.
- We consider constant background values b for the twisted scalars and consider:



- The disk diagram represented above acts as a classical source for Φ_{open}, which acquires a non-trivial profile in the plane transverse to the defect.

From worldsheet correlators to profiles

- The disk diagram acts as a classical source for $\Phi_{\rm open}$:

$$\Box \Phi_{\text{open}}(x) = J(x)$$

- The open/closed coupling $\langle \mathcal{V}_{\text{open}}(k) \rangle_{b,l}$ is the analog of $\widetilde{J}(k)$. The profile is obtained by inverting the equation:

$$\Phi_{\text{open}}(x) = \int \frac{d^4 k}{(2\pi)^2} \, \frac{e^{ik\cdot X}}{k^2} \langle \mathcal{V}_{\text{open}}(k) \rangle_{b,l} \; .$$

- The background value b will be identified with the parameters that appear in the profiles of the Gukov-Witten defect.

Comments

- This is similar to how the boundary state formalism was used to derive the leading correction to the p-brane profile, when considered as a soliton.
  di Vecchia, Frau, Lerda, Pesando, Russo, Sciuto, '97

- This work is also motivated by earlier work on deriving the classical gauge instanton profile using worldsheet methods.
  Billo, Frau, Fucito, Lerda, Liccardo, Pesando, '02

- These examples share the same idea: a worldsheet with either closed or open string insertions acts as a source (one-point function) for a particular field.

- In our case, the relevant worldsheet is the disk with the insertion of a twisted closed string field.

What we do:

- Schematically, we will calculate

$$\langle \mathcal{V}_{\text{closed}}(z,\bar{z})\, \mathcal{V}_{\text{open}}(x) \rangle_{\text{D3}_I}$$

- **Twisted closed string states** of the orbifold, and their vertex operators

$$\mathcal{V}_{\text{closed}}(z,\bar{z}) = \mathcal{V}_L(z)\, \widetilde{\mathcal{V}}_R(\bar{z}) \ .$$

- The presence of the boundary leads to reflection rules that relate the left and right-movers. This can be read off from the **boundary state**.
- **Open string spectrum** on the D3 branes (new results).
- We will calculate the open/closed coupling using standard methods of string perturbation theory.

Closed string theory on the orbifold: twisted sectors

- We consider type IIB string theory on the orbifold
  Anselmi et al. '93, Douglas-Moore '96

  $$\mathbb{C}_{(1)} \times \frac{\mathbb{C}_{(2)} \times \mathbb{C}_{(3)}}{\mathbb{Z}_2} \times \mathbb{C}_{(4)} \times \mathbb{C}_{(5)} \; .$$

  Our goal is to write vertex operators for the twisted sector scalars.

- The isometry of $\mathbb{C}_{(2)} \times \mathbb{C}_{(3)}$ is $\mathrm{SU}(2)_+ \times \mathrm{SU}(2)_- \sim \mathrm{SO}(4)$. The orbifold group is embedded into $\mathrm{SU}(2)_-$ and so all the physical states are representations of $\mathrm{SU}(2)_+$.

- In the bosonic sector:

  $$Z^2(\mathrm{e}^{2\pi\mathrm{i}}\, z) = -\, Z^2(z) \qquad \text{and} \qquad Z^3(\mathrm{e}^{2\pi\mathrm{i}}\, z) = -\, Z^3(z) \; .$$

  The twist operator $\Delta(z)$, with $h = \frac{1}{4}$ creates the vacuum for these twisted bosonic fields.

- For the fermions, the modding of the fields is shifted by $1/2$ in the twisted sector: so NS sector fields have zero-modes in the (z^2, z^3) directions.

Vertex operators

The NS/R sector vertex operators are made up of the following contributions:

- **Bosons**: twisted sector ground state Δ, which has $h = \frac{1}{4}$.
- **Fermions**: in the $\mathbb{Z}_2$ case, these correspond to spin fields in both the NS and R sectors.

  The spin fields in the NS sector correspond to spinors of SO(4), while the spin fields of the R sector correspond to spinors of SO(6).
- **Momentum**: we will set $k = 0$ and have constant background values for the twisted scalars.
- **Super-ghosts**: $e^{-\ell\phi}$. For the NS sector, we choose the (-1) picture while for the R sector, we choose either the $(-1/2)$ picture or the dual $(-3/2)$ picture.

Closed string theory on the orbifold

- In the NS sector, we have the following vertex operators:

  $$\mathcal{V}^{\alpha} = \mathrm{e}^{-\phi(z)}\,\Delta(z)\,S^{\alpha}(z)\ .$$

  These have conformal dimension $\frac{1}{2}+\frac{1}{4}+\frac{1}{4}=1$.
  The vertex operators in the right-moving sector can be similarly written.

- The closed string states are obtained by combining the left-movers and right-movers. The four independent components can be decomposed into a real scalar b and a triplet b_c (with $c=1,2,3$).

  $$\begin{aligned} b &\longleftrightarrow \mathcal{V}_b(z,\bar{z}) = \mathrm{i}\,\epsilon_{\alpha\beta}\,\mathcal{V}^{\alpha}(z)\,\widetilde{\mathcal{V}}^{\beta}(\bar{z})\ , \\ b_c &\longleftrightarrow \mathcal{V}_{b_c}(z,\bar{z}) = (\epsilon\,\tau_c)_{\alpha\beta}\,\mathcal{V}^{\alpha}(z)\,\widetilde{\mathcal{V}}^{\beta}(\bar{z})\ , \end{aligned}$$

  where τ_c are the usual Pauli matrices. They transform in the $(\mathbf{1},\mathbf{1})$ and $(\mathbf{3},\mathbf{1})$ representations of $\mathrm{SU}(2)_+ \times \mathrm{SU}(2)_- \sim \mathrm{SO}(4)$.

Closed string theory on the orbifold

- In the R sector, the fermions have zero modes only in the complex $(1, 4, 5)$ directions. So the vacuum corresponds to chiral and anti-chiral spinors in the 6d Euclidean space:

$$\mathcal{V}^{A}(z) = \Delta(z)\, S^{A}(z)\, \mathrm{e}^{-\frac{1}{2}\phi(z)} ,$$
$$\mathcal{V}^{\dot{A}}(z) = \Delta(z)\, S^{\dot{A}}(z)\, \mathrm{e}^{-\frac{3}{2}\phi(z)} .$$

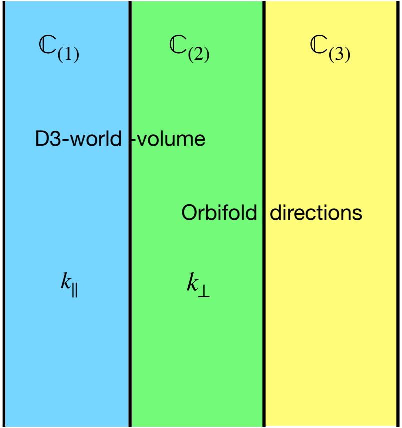
  These have dimensions $\frac{1}{4} + \frac{3}{8} + \frac{3}{8} = 1$. One can write down similar operators in the right-moving sector. The picture number and chirality are related by the GSO projection.

- In the asymmetric $(-\frac{1}{2}, -\frac{3}{2})$-superghost picture the vertex operators $\mathcal{V}^{A}(z)\, \widetilde{\mathcal{V}}^{\dot{B}}(\bar{z})$ describe R/R potentials, which have sixteen independent components. These can be decomposed into a scalar c and an anti-symmetric tensor c_{MN} of SO(6):

$$c \quad \longleftrightarrow \quad \mathcal{V}_{c}(z, \bar{z}) = C_{A\dot{B}}\, \mathcal{V}^{A}(z)\, \widetilde{\mathcal{V}}^{\dot{B}}(\bar{z}) ,$$
$$c_{MN} \quad \longleftrightarrow \quad \mathcal{V}_{c_{MN}}(z, \bar{z}) = (C\, \Gamma_{MN})_{A\dot{B}}\, \mathcal{V}^{A}(z)\, \widetilde{\mathcal{V}}^{\dot{B}}(\bar{z}) .$$

Adding D3-branes

- The D3-branes we add are extended along the (z^1, z^2) directions while the orbifold action is along the (z^2, z^3) directions.



Decomposing the closed string fields

- The presence of the D3-brane breaks the isometry further.
- The closed string fields decompose into representations of this unbroken symmetry group. In the NS/NS sector, the triplet b_c decomposes into a real scalar and a pair of complex conjugate scalars:

$$\begin{aligned} b \quad &\longleftrightarrow \quad \mathcal{V}_b(z,\bar{z}) = \mathrm{i}\,\epsilon_{\alpha\beta}\,\mathcal{V}^{\alpha}(z)\,\widetilde{V}^{\beta}(\bar{z})\ , \\ b' \quad &\longleftrightarrow \quad \mathcal{V}_{b'}(z,\bar{z}) = (\epsilon\tau_3)_{\alpha\beta}\,\mathcal{V}^{\alpha}(z)\,\widetilde{V}^{\beta}(\bar{z})\ , \\ b_{\pm} \quad &\longleftrightarrow \quad \mathcal{V}_{b_{\pm}}(z,\bar{z}) = (\epsilon\tau_{\pm})_{\alpha\beta}\,\mathcal{V}^{\alpha}(z)\,\widetilde{V}^{\beta}(\bar{z}) \end{aligned}$$

  where $\tau_{\pm} = (\tau_1 \pm \mathrm{i}\,\tau_2)/2$.

- In the R/R sector, the scalar c remains. From the anti-symmetric tensor c_{MN}, we obtain another scalar c':

$$\begin{aligned} c \quad &\longleftrightarrow \quad \mathcal{V}_c(z,\bar{z}) = C_{A\dot{B}}\,\mathcal{V}^{A}(z)\,\widetilde{\mathcal{V}}^{\dot{B}}(\bar{z})\ , \\ c' \quad &\longleftrightarrow \quad \mathcal{V}_{c'}(z,\bar{z}) = (C\,\Gamma_{12})_{A\dot{B}}\,\mathcal{V}^{A}(z)\,\widetilde{\mathcal{V}}^{\dot{B}}(\bar{z})\ . \end{aligned}$$

Boundary States

- The boundary state $|\mathrm{D3}; I\rangle$ corresponding to the Ith fractional D3-brane encodes the couplings to the closed string states (via 1-pt. functions).
- They are labelled by irreps of the orbifold group; so for $\mathbb{Z}_2$, there are two types of fractional branes.

$$|\mathrm{D3}; I\rangle = \mathcal{N}\,|\mathrm{U}\rangle + \mathcal{N}'\,|\mathrm{T}; I\rangle \quad \text{with} \quad |\mathrm{T}; I\rangle = (-1)^I\,|\mathrm{T}\rangle\ .$$

- The twisted scalars are ground states and so only the zero-mode part of the boundary state turns out to be relevant.
- From the form of the boundary state one derives **reflection rules** that relate the left and right moving parts of the closed string vertex operators:

$$\begin{aligned} \widetilde{\mathcal{V}}^{\alpha}(\bar{z}) &\longrightarrow (-1)^I(\gamma_4\gamma_3)^{\alpha}{}_{\beta}\,\mathcal{V}^{\beta}(\bar{z})\ , \\ \widetilde{\mathcal{V}}^{\dot{A}}(\bar{z}) &\longrightarrow (-1)^I(\Gamma_1\Gamma_2)^{\dot{A}}{}_{\dot{B}}\,\mathcal{V}^{\dot{B}}(\bar{z})\ . \end{aligned}$$

One uses this in combination with the doubling trick.

Open Strings

- For D-branes at orbifolds, the open strings are the invariant combinations of

  $$\text{oscillators} \times \text{Chan-Paton factors}$$

- But for these orbifolds, in which the brane world-volume extends along the orbifold directions, there is an additional factor:

  $$\text{oscillators} \times \text{Chan-Paton factors} \times \text{momentum factors}$$

- In the $\mathbb{Z}_2$ case, this is simple to see:

  $$\cos(\kappa_\perp Z_\perp) \quad \text{and} \quad \mathrm{i}\,\sin(\kappa_\perp Z_\perp)\ .$$

  have charges 0 and 1 respectively under the orbifold action.

- In the $\mathbb{Z}_M$ case:

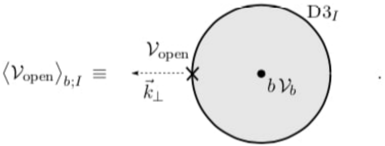
  $$\mathcal{E}_I = \frac{1}{M}\sum_{J=0}^{M-1} \omega^{-IJ}\, \mathrm{e}^{\mathrm{i}\,(\omega^{-J}\kappa_\perp \bar{Z}_\perp + \omega^{J}\bar{\kappa}_\perp Z_\perp)}\ .$$

Open Strings

- Net result: no open string state is projected out if it carries momentum along the directions along the orbifold.

- The 1-loop partition function also supports this interpretation as it shows two sectors: a 2d sector and a 4d sector. So, we interpret this as a surface defect in $\mathcal{N} = 4$ SYM.

- We are interested in the following correlator:



- So, only the diagonal CP factors will be of relevance for this open/closed correlator.

Open Strings

- Usually, in the canonical (-1) picture, these are of the form

$$e^{-\phi}\,\psi^{\mu}\,e^{\mathrm{i}\,k\cdot X} \quad \text{or} \quad e^{-\phi}\,\psi^{a}\,e^{\mathrm{i}\,k\cdot X}$$

- One could just as well, work in the (0)-picture by acting with a PCO:

$$(\partial X^{\mu} + \mathrm{i}\,(k\cdot\psi)\psi^{\mu})e^{\mathrm{i}\,k\cdot X}\ .$$

- Now, we start with a parity even plane wave in the (-1) picture for states with diagonal CP factors:

$$\begin{aligned} &e^{-\phi}\,\Psi^{\|}\,\cos(\kappa_{\perp}\cdot Z_{\perp})\,e^{\mathrm{i}\kappa_{\|}\cdot Z_{\|}} \quad \text{for} \quad \| \in \{1,\bar{1}\} \\ &e^{-\phi}\,\Psi^{\perp}\,\mathrm{i}\,\sin(\kappa_{\perp}\cdot Z_{\perp})e^{\mathrm{i}\kappa_{\|}\cdot Z_{\|}} \quad \text{for} \quad \perp \in \{2,\bar{2}\} \end{aligned}$$

  and so on. Acting with the PCO can give rise to vertex operators in the (0) picture.

Vertex Operators

The closed string vertex operators are in the $(-1,-1)$ picture. We thus need the open string vertex operators in the 0-picture.

$$A_1 \longrightarrow \mathcal{V}_{A_1} = \Big[\big(\mathrm{i}\,\partial Z^1 + \kappa_\parallel \cdot \Psi_\parallel\, \Psi^1\big)\cos(\kappa_\perp \cdot Z_\perp) + \mathrm{i}\,\kappa_\perp \cdot \Psi_\perp\, \Psi^1\, \sin(\kappa_\perp \cdot Z_\perp)\Big]\mathrm{e}^{\mathrm{i}\,\kappa_\parallel \cdot Z_\parallel}\ .$$

$$A_2 \longrightarrow \mathcal{V}_{A_2} = \Big[\big(\mathrm{i}\,\partial Z^2 + \kappa_\parallel \cdot \Psi_\parallel\, \Psi^2\big)\,\mathrm{i}\,\sin(\kappa_\perp \cdot Z_\perp) + \kappa_\perp \cdot \Psi_\perp\, \Psi^2\, \cos(\kappa_\perp \cdot Z_\perp)\Big]\mathrm{e}^{\mathrm{i}\,\kappa_\parallel \cdot Z_\parallel}\ .$$

Here we use a complex notation for the gauge field

$$\mathbf{A} = A \cdot dx = A_1\, d\bar{w} + \bar{A}_1\, dw + A_2\, d\bar{z} + \bar{A}_2\, dz\ .$$

We now have all the ingredients to calculate the open/closed correlators in string theory of the form

$$\big\langle \mathcal{V}_{\text{closed}}(z,\bar{z})\, \mathcal{V}_{\text{open}}(x) \big\rangle_I$$

Open/closed correlators: some technicalities

- To illustrate the methods let us calculate the coupling between A_2 and b. The open/closed coupling is given by

$$\left\langle \mathcal{V}_{A_2} \right\rangle_{b;l} = \int \frac{dz\, d\bar{z}\, dx}{dV_{\text{proj}}} \left\langle b\, \mathcal{V}_b(z,\bar{z})\, \mathcal{V}_{A_2}(x) \right\rangle_l$$

- The vertex operators are:

$$\mathcal{V}_{A_2} = \Big[\big(\mathrm{i}\,\partial Z^2 + \kappa_{\parallel}\cdot\Psi_{\parallel}\,\Psi^2\big)\,\mathrm{i}\,\sin(\kappa_{\perp}\cdot Z_{\perp}) + \kappa_{\perp}\cdot\Psi_{\perp}\,\Psi^2\,\cos(\kappa_{\perp}\cdot Z_{\perp})\Big]\mathrm{e}^{\mathrm{i}\,\kappa_{\parallel}\cdot Z_{\parallel}}\ ,$$

$$b\,\mathcal{V}_b(z,\bar{z}) = \mathrm{i}\, b\, \epsilon_{\alpha\beta}\, \mathcal{V}^{\alpha}(z)\, \widetilde{V}^{\beta}(\bar{z})\ .$$

with the zero-momentum vertex operator given by:

$$\mathcal{V}^{\alpha} = \mathrm{e}^{-\phi(z)}\, \Delta(z)\, S^{\alpha}(z)\ .$$

- To compute the correlator we use the reflection rule:

$$\begin{aligned} \left\langle \mathcal{V}_b(z,\bar{z})\, \mathcal{V}_{A_2}(x) \right\rangle_l &= \mathrm{i}\, \epsilon_{\alpha\beta} \left\langle \mathcal{V}^{\alpha}(z)\, \widetilde{\mathcal{V}}^{\beta}(\bar{z})\, \mathcal{V}_{A_2}(x) \right\rangle_l \\ &= (-1)^l\, \mathrm{i}\, \epsilon_{\alpha\beta} (\gamma_4\gamma_3)^{\beta}{}_{\gamma} \left\langle \mathcal{V}^{\alpha}(z)\, \mathcal{V}^{\gamma}(\bar{z})\, \mathcal{V}_{A_2}(x) \right\rangle \end{aligned}$$

The basic correlators

The calculation reduces to CFT correlators in a product of free field theories:

$$\left\langle \mathrm{e}^{-\phi(z)}\,\mathrm{e}^{-\phi(\bar{z})} \right\rangle = \frac{1}{z-\bar{z}} \ ,$$

$$\left\langle \mathrm{e}^{\mathrm{i}\,\kappa_{\parallel}\cdot Z_{\parallel}(x)} \right\rangle = \delta^{(2)}(\kappa_{\parallel}) \ ,$$

$$\left\langle \Delta(z)\Delta(\bar{z})\cos(k_{\perp}\cdot Z_{\perp})(x) \right\rangle = \frac{1}{(z-\bar{z})^{\frac{1}{2}}} \ ,$$

$$\left\langle S^{\alpha}(z)S^{\gamma}(\bar{z})\,\psi_m(x)\psi_n(x) \right\rangle = \frac{1}{2}\,\frac{(\gamma_n\gamma_m\widehat{C}^{-1})^{\alpha\gamma}}{(z-\bar{z})^{-\frac{1}{2}}(z-x)(\bar{z}-x)} \ .$$

Putting everything together, we obtain

$$\left\langle \mathcal{V}^{\alpha}(z)\,\mathcal{V}^{\gamma}(\bar{z})\,\mathcal{V}_{A_2}(x) \right\rangle = \mathrm{i}\,\frac{\kappa_2}{2}\,\frac{(\gamma_4\gamma_3\widehat{C}^{-1})^{\alpha\gamma}}{(z-\bar{z})(z-x)(\bar{z}-x)}\,\delta^{(2)}(\kappa_{\parallel}) \ .$$

Summary of open/closed correlators

After some γ-matrix algebra, one can extract the string amplitude:

$$\big\langle \mathcal{V}_{A_2} \big\rangle_{b;I} = (-1)^{I+1}\, b\, \kappa_2\, \delta^{(2)}(\kappa_{\parallel}) \ .$$

The other non-zero couplings are the following:

$$\big\langle \mathcal{V}_{\Phi} \big\rangle_{b_+;I} = (-1)^{I+1}\, \mathrm{i}\, b_+\, \bar{\kappa}_2\, \delta^{(2)}(\kappa_{\parallel})$$
$$\big\langle \mathcal{V}_{A_1} \big\rangle_{c;I} = (-1)^{I+1}\, 2\, \mathrm{i}\, c\, \kappa_1\, \delta^{(2)}(\kappa_{\parallel}) \ .$$

Analogous correlators can be obtained for the complex conjugate fields.

- None of the open string fields couples to b' or c'.
- The momentum conserving delta-function is only along the $\mathbb{C}_{(1)}$-plane directions.
- It now remains to read the profiles of the open string fields in position space from these correlators.

Profiles from Fourier transforms

- The non-zero correlators indicate that constant background values for certain twisted scalars lead to tadpoles for the open string fields:

$$\Box \Phi_{\text{open}}(x) = J(x)$$

- The open/closed coupling we have calculated is the analog of $\widetilde{J}(k)$. The profile is obtained by inverting the equation:

$$\Phi_{\text{open}}(x) = \int \frac{d^4k}{(2\pi)^2}\, e^{ik\cdot x}\, \frac{\widetilde{J}(k)}{k^2}\ .$$

- From the open/closed correlators, $\widetilde{J}(k) = \kappa_2\, \delta^2(\kappa_{\|})$ or its complex conjugate. The Fourier transform is effectively two dimensional.

$$\int d^2\kappa_\perp\, e^{i\kappa_\perp\cdot z}\, \frac{\kappa_2}{\kappa_\perp^2} \sim \partial_{\bar z} \log(z\bar z) \sim \frac{1}{\bar z}$$

This is simply the derivative of the free propagator in two dimensions.

Profiles from Fourier transforms

- Applying the above procedure to the profile of its gauge field A_2, we find:

  $$A_{2,l} = (-1)^{l+1} \frac{\mathrm{i}\, b}{4\pi \bar{z}} \ .$$

  We see that it is induced only by the background vev for the singlet b.

- Combining this with the corresponding result for the complex conjugate field, the gauge field acquires the following profile

  $$\mathbf{A}_l = A_{2,l}\, d\bar{z} + \bar{A}_{2,l}\, dz = (-)^{l+1} \frac{\mathrm{i}\, b}{4\pi} \left(\frac{d\bar{z}}{\bar{z}} - \frac{dz}{z} \right) = (-)^{l+1} \frac{b}{2\pi}\, d\theta$$

  where θ is the polar angle in the $\mathbb{C}_{(2)}$ plane transverse to the defect.

- One can do a similar analysis for the scalar profiles:

  $$\Phi_l = (-)^l \frac{b_+}{4\pi z} \ .$$

Profiles from Fourier transforms

- For n_0 fractional D3-branes of type 0 and n_1 fractional D3-branes of type 1, which describes a gauge theory with group $\mathrm{U}(n_0+n_1)$ broken to the Levi group $\mathrm{U}(n_0)\times\mathrm{U}(n_1)$, we thus have:

$$\mathbf{A} = -\frac{b}{2\pi}\begin{pmatrix}\mathbb{I}_{n_0} & 0 \\ 0 & -\mathbb{I}_{n_1}\end{pmatrix} d\theta \ ,$$

$$\Phi = \frac{b_+}{4\pi}\begin{pmatrix}\mathbb{I}_{n_0} & 0 \\ 0 & -\mathbb{I}_{n_1}\end{pmatrix} \frac{1}{z} \ .$$

These are precisely the expected singular profiles for a monodromy defect of GW type $[n_0, n_1]$.

The 2d θ-term

- Let us recall the coupling to the RR scalar:

  $$\left\langle \mathcal{V}_{A_1} \right\rangle_{c;l} = (-1)^{l+1}\, 2\,\mathrm{i}\, c\, \kappa_1\, \delta^{(2)}(\kappa_\|)\ .$$

  Naively this looks to be zero due to the delta function. So, this coupling does not indicate a source for the $A_\|$.

- Putting back in the corresponding polarizations of the gauge field, the resulting sum can be interpreted as an effective interaction term involving the gauge field strength in the longitudinal directions.

  $$\bar{A}_1 \left\langle \mathcal{V}_{A_1} \right\rangle_{c;0} + A_1 \left\langle \mathcal{V}_{\bar{A}_1} \right\rangle_{c;0} = -2\,\mathrm{i}\, c\, (\kappa_1\, \bar{A}_1 - \bar{\kappa}_1\, A_1)\, \delta^{(2)}(k_\|) = 2\,\mathrm{i}\, c\, \widetilde{F}_0\, \delta^{(2)}(\kappa_\|)$$

- The Fourier transform of the amplitude is

  $$\mathrm{i}\, c \int d^2 k_\|\, \widetilde{F}_0\, \delta^{(2)}(\kappa_\|) \times 2\, \delta^{(2)}(z_\perp) = \frac{\mathrm{i}\, c}{2\pi} \int d^2 x_\|\, F_0 \times 2\, \delta^{(2)}(z_\perp)$$

  where F_0 is the field strength in configuration space. More generally, if the gauge field strength has a non-vanishing first Chern class along the defect D, then we can be interpret this as an effective interaction term localized on the defect D.

The 2d θ-term

- So in the path-integral of the underlying gauge theory, if $c \neq 0$, one has the following phase factor:

$$\exp\left(\frac{\mathrm{i}\,c}{2\pi}\int_D F_0\right) .$$

- If we extend this argument to a system made of n_0 fractional D3-branes of type 0 and n_1 fractional D3-branes of type 1, the phase factor becomes

$$\exp\left(\mathrm{i}\sum_I (-1)^I \frac{c}{2\pi}\int_D \mathrm{Tr}_{\,\mathrm{U}(n_I)} F_I\right)$$

  which has exactly the same form of the one of the GW monodromy defect with

$$\eta_I = (-1)^I \frac{c}{2\pi} .$$

- The continuous data of the GW surface defect are encoded in the expectation values of the closed string fields in the orbifold twisted sectors according to

$$\{\alpha_I, \beta_I, \gamma_I, \eta_I\} = \left\{(-1)^{I+1}\frac{b}{2\pi}, (-1)^I \frac{\mathrm{Re}(b_+)}{2\pi}, (-1)^I \frac{\mathrm{Im}(b_+)}{2\pi}, (-1)^I \frac{c}{2\pi}\right\} .$$

Consistency with S-duality

- The orbifold background is the zero-size limit of a smooth ALE space. From this pespective, all the twisted sector scalars can be given a geometric interpretation.

- The NS/NS singlet b and the R/R scalar c correspond to moduli obtained by integrating the $(B_{(2)}, C_{(2)})$ fields over the vanishing $\mathbb{P}^1$.

  $$b = \int_{\omega_2} B_{(2)} \ , \qquad c = \int_{\omega_2} C_{(2)} \ .$$

  The parameters α_I and η_I can be written in the following suggestive way

  $$\alpha_I = \frac{(-1)^{I+1}}{2\pi} \int_{\omega_2} B_{(2)} \ , \qquad \eta_I = \frac{(-1)^{I}}{2\pi} \int_{\omega_2} C_{(2)} \ .$$

- The S-duality transformations of b and c are thus inherited from those of the parent Type II B fields. Under a duality transform $\Lambda = \left(\begin{smallmatrix} m & n \\ p & q \end{smallmatrix}\right) \in \mathrm{SL}(2, \mathbb{Z})$:

  $$\begin{pmatrix} C_{(2)} \\ B_{(2)} \end{pmatrix} \longrightarrow \begin{pmatrix} m & n \\ p & q \end{pmatrix} \begin{pmatrix} C_{(2)} \\ B_{(2)} \end{pmatrix}$$

  One can check that this exactly reproduces the transformation of (α_I, η_I) under S-duality proposed by Gukov-Witten.

Consistency with S-duality

- Similarly, $b_\pm$ are identified with complex structure moduli of the ALE space and they are metric moduli.
- The string-frame metric $G_{\mu\nu}$ transforms as

  $$G_{\mu\nu} \longrightarrow |p\tau + q|\, G_{\mu\nu}$$

  under an $\mathrm{SL}(2,\mathbb{Z})$ transformation, where τ is the axio-dilaton field.

  This is precisely the transformation of the (β,γ) parameters of the surface defect if we identify τ with the complexified gauge field coupling.
- This is a check on our proposal for the identification of the continuous parameters of the surface defect:

  $$\{\alpha_I,\beta_I,\gamma_I,\eta_I\} = \left\{(-1)^{I+1}\frac{b}{2\pi},(-1)^I\frac{\mathrm{Re}(b_+)}{2\pi},(-1)^I\frac{\mathrm{Im}(b_+)}{2\pi},(-1)^I\frac{c}{2\pi}\right\}.$$

Comments on the $\mathbb{Z}_M$ orbifold

- The generic surface defect is described by a partition $[n_0, n_1, \ldots n_{M-1}]$ and $4M$ continuous parameters $\{\alpha_I, \beta_I, \gamma_I, \eta_I\}$. These are realized by fractional D3-branes on a $\mathbb{Z}_M$ orbifold.
  SA, Billo, Frau, Lerda, Mahato, Part 2

- The conceptual ideas go through exactly as in the $\mathbb{Z}_2$ case though there are some subtleties.

- Closed strings: the twisted sector scalars are now excited states (no spin-field description).

- Open strings: the sin and cos functions are generalized to linear combinations of plane waves that transform covariantly under the orbifold action.

- Identification is more involved:

$$\alpha_I = -\frac{1}{2\pi} \sum_{a=1}^{M-1} \sin\left(\frac{\pi a}{M}\right) \omega^{-I\,a}\, b_{\mathrm{s}}^{(a)} \ .$$

Final remarks

- We have engineered Gukov-Witten defects using fractional D3-branes on a $\mathbb{C}^2/\mathbb{Z}_M$ orbifold in type IIB string theory.
- The background values of twisted scalars of the orbifold induces one-point functions for the open string fields.
- We could recover the singular profiles of the fields in the gauge theory from worldsheet correlators.
- This gives us a framework where questions about surface defects can be addressed within perturbative string theory.

Much to explore . . .