Scale vs Conformal from the viewpoint of (topological) twist

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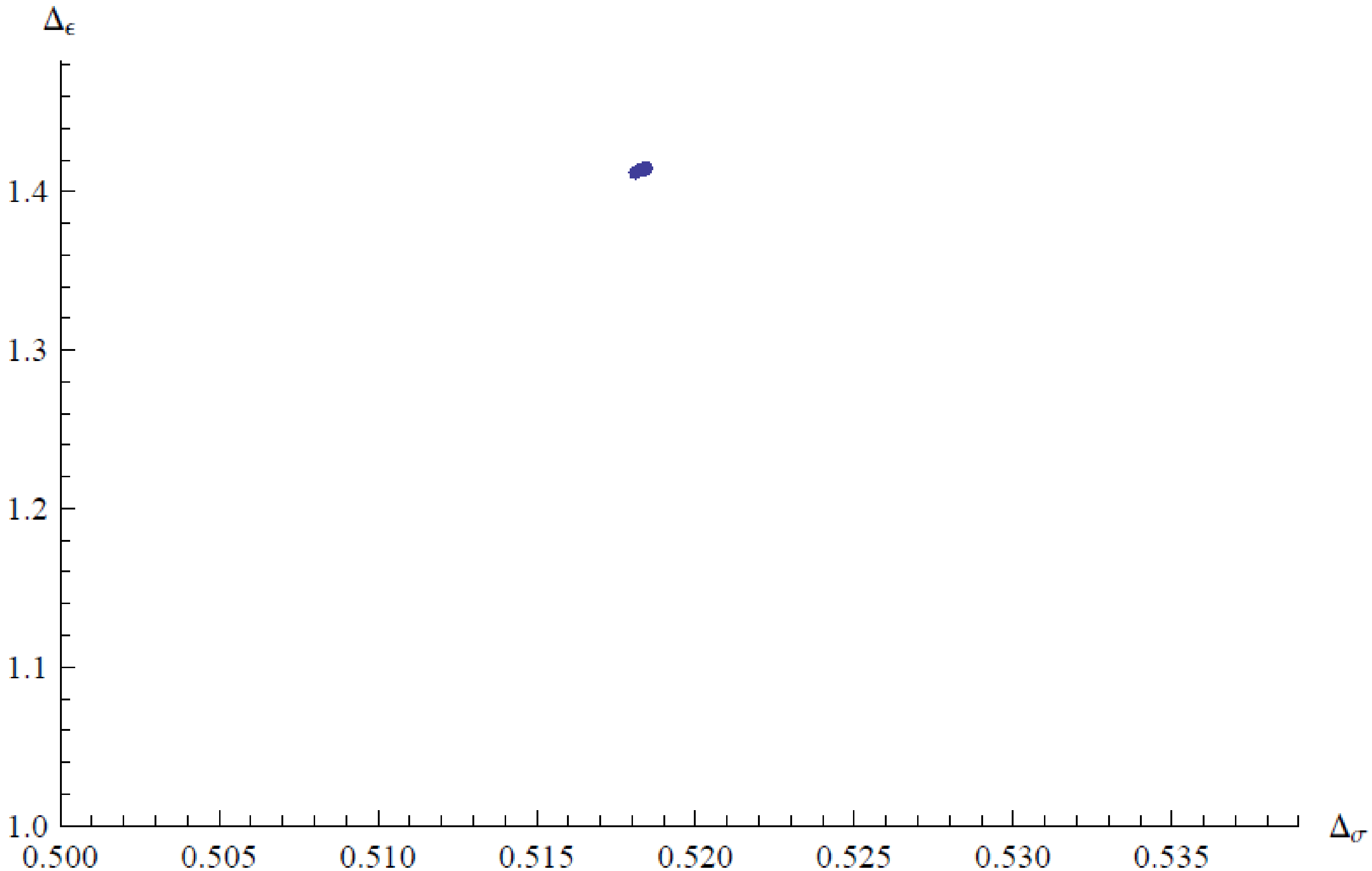
1608.02651, 1611.10040

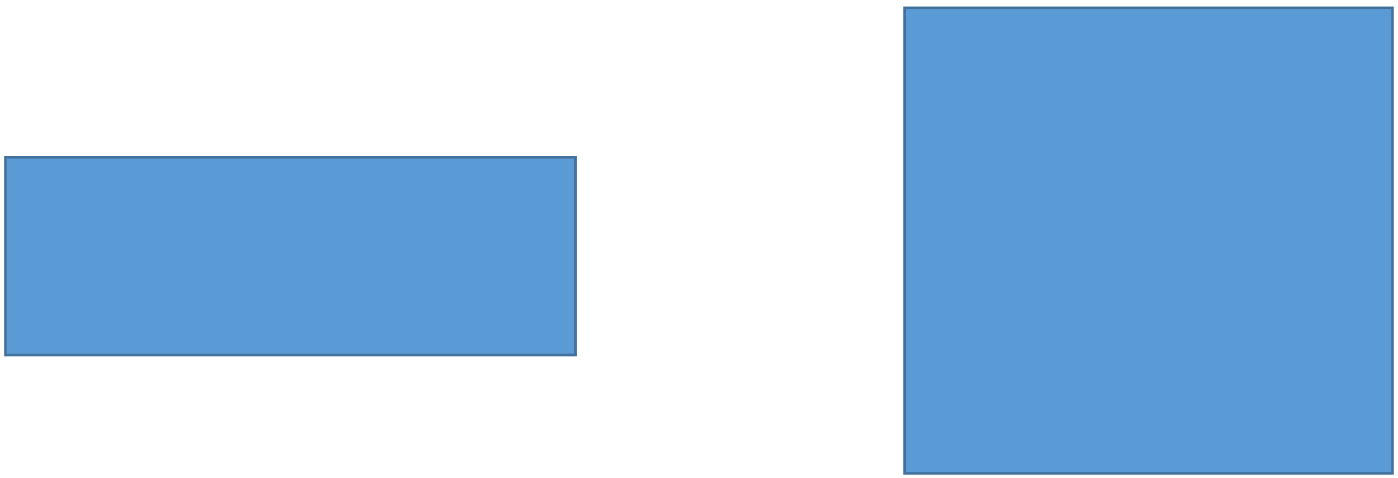
also more recent discussions with S. Rychkov (to appear)

Conformal bootstrap is powerful



Critical exponents in 3d Ising computed on my laptop

But rectangles are not (always) squares



Actually, in Japanese elementary schools rectangles are (by definition) never squares…

Scale vs Conformal

- Conformal invariance is very powerful (e.g. conformal bootstrap) in critical phenomena
- Critical phenomena → Scale invariance

- Gap between scale invariance and conformal invariance?
- I've been interested for a while.
- I wrote a review: 1302.0884

- (2,0) vs (1,1) in 6d. No-fixed point???

Scale vs Conformal

- Consider a local relativistic (Euclidean) QFT with (symmetric) energy-momentum tensor $T_{\mu\nu}$

$$T_{\mu\nu} = T_{\nu\mu} \qquad \partial^\mu T_{\mu\nu} = 0$$

- Scale invariance

$$T^\mu_\mu = \partial^\mu J_\mu \neq \Box O \qquad J_\mu$$:virial current

$$\rightarrow \quad D^\mu = x_\nu T^{\mu\nu} - J^\mu$$

- Conformal

$$T^\mu_\mu = \Box O \sim 0$$

A simple counter-example (El-Shawk Nakayama Rychkov)

- Consider free U(1) gauge theory in d-dimension

$$S = \int d^d x F_{\mu\nu} F^{\mu\nu}$$

- Massless → obviously scale invariant
- But it is not conformal

$$T^{\mu}_{\mu} = (d-4) F_{\mu\nu} F^{\mu\nu} = \frac{d-4}{2} \partial^{\mu} (F_{\mu\nu} A^{\nu})$$

- It is not (so) easy to justify conformal invariance in higher dim gauge theory fixed point (e.g. 6d (1,1))

Argument for scale → conformal?

- Genericity argument

$$T^\mu_\mu = \partial^\mu J_\mu \neq \Box O$$

  - Requires dimension d-1 vector operator
  - Generically most operators get renormalization (anomalous dimensions)
  - Looks very fine-tuned and unlikely
- Unitarity argument
  - This is the proof by Zamoldochkov and Polchinski in d=2 (or d=4 by Schwimmer, Komargodski etc)
  - If scale without conformal, c (or a) decreases forever → in unitary theory, $T^\mu_\mu = \partial^\mu J_\mu \neq \Box O$ is essentially (on-shell) zero

Genericity argument??

- I have a long-standing debate with Slava about the genericity argument (see his EPFL lecture note)
- From the beginning, scale ⊂ conformal, so why is it harder to get scale (if no assumption is made)?
- Rather, if scale only, dim of J must be AUTOMATICALLY d-1, and it should not be fine-tuning (but Salava complains these two are apparently different conditions)
- So I constructed many models, but Slava had not been satisfied…

Scale without Conformal from (topological) twist

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(Topological) twist

- Consider d-dim CFT with O(d) internal symmetry
- Define twisted Euclidean rotation by O(d) in the product of O(d) x O(d)
- Lorentz spin and internal spin mixed
  - E.g. A_μ^I becomes tensor + scalar
- Unitarity (reflection positivity) is lost...
- Spinor-supersymmetry → scalar-supersymmetry

Twist and conformal symmetry

- In curved background we add extra connection

$$\delta S = \int d^d x \sqrt{g} \omega_\mu^{IJ} J^\mu_{IJ}$$

- → Energy-momentum tensor is modified

$$\tilde{T}_{\mu\nu} = T_{\mu\nu} + \frac{1}{2}(\partial_\rho J_\mu^{[IJ]} \delta_{I\nu} \delta_J^\rho + \partial_\rho J_\nu^{[IJ]} \delta_{I\mu} \delta_J^\rho)$$

- Special conformal symmetry appears to be lost

$$\tilde{T}_\mu^\mu = \partial_\rho J_\mu^{[IJ]} \delta_{I\mu} \delta_J^\rho$$

- Note, however, we did not do anything in flat space (→ "special conformal symmetry" is hidden)

Adding twisted deformations

- We have new "twisted scalar deformations"!

$$O_I^{\mu} \to O_I^I \ , \quad O_{\mu\nu}^{IJ} \to O_{IJ}^{IJ}$$

→ twisted RG flow (not studied very much)

- Novel fixed points → would be scale without conformal
- Perturbatively it must be so because small RG effects cannot get rid of virial current

$$\tilde{T}_{\mu}^{\mu} = \partial_{\rho} J_{\mu}^{[IJ]} \delta_{I\mu} \delta_{J}^{\rho} + \mathcal{O}(g)$$

- With the deformation, one cannot undone the twist!
- Quite general. Argument against genericity?

Twisted SFT Examples

1. M-theory
2. Perturbative (discussions with Rychkov, to appear)
3. in nature?

Holography 1: effective description

- We work on AdS4/CFT3
- Einstein + SU(2) Yang-Mills (in Euclidean signature)

$$S = \int d^4x \sqrt{g}(R - 2\Lambda - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a)$$

- Twisted deformation with novel fixed points → would be scale w/o conformal
- ansatz

$$ds^2 = f(z)\frac{dz^2 + dx_i^2}{z^2} \qquad A^a = g(z)dx^i\delta^a_i$$

- Scale (w/o conformal) solution

$$f(z) = 1 \ , \quad g(z) = \frac{1}{z}$$

Holography 2: M-theory uplift

- We work on AdS4/CFT3
- Consider (Euclidean) M-theory compactification on AdS4 x S7
- Pope argued Einstein + cc + SU(2) YM is consistent truncation in Lorentzian signature
- Consistent truncation works in Euclidean signature as well although the d=11 flux is now pure imaginary
- (Consistency of the truncation is "miraculous")

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + g_{mn}(y)(dy^m - K^{am}(y)A^a_\mu(x)dx^\mu)(dy^n - K^{bn}(y)A^b_\nu dx^\nu) \ ,$$

$$F_{\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\delta}} = 3im\epsilon_{\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\delta}}$$

$$F_{\bar{\alpha}\bar{\beta}\bar{c}\bar{d}} = -\frac{i}{2}\tilde{G}^a_{\bar{\alpha}\bar{\beta}}M^a_{\bar{c}\bar{d}} \ ,$$

Holography 3: interpretation

- One can construct RG-flow solution

$$ds^2 = \frac{dz^2 + dx_i^2}{z^2} \;, A_i^a = \frac{1}{z+z_0} dx^i \delta_i^a$$

- UV: ABJM (BLG) → deformed by dim 2 twisted scalar → IR scale w/o conformal

- In field theory, deformation is induced by topologically twisted SU(2) R-current (out of SO(8) R-symmetry)

- Chern-Simons "like" deformation

$$O = J_\mu^{IJ} \epsilon_{IJ}^\mu = \Phi^I \partial_\mu \Phi^J \epsilon_{IJ}^\mu + \cdots$$

- Field theory analysis would be difficult, but the gravity side is surprisingly simple and universal.

QFT construction 1

- Consider O(d) model $$S = \int d^d x \partial_\mu \Phi^I \partial^\mu \Phi_I$$

Twist → free vector in Feynman gauge

$$S = \int d^d x \partial_\mu \Phi^\nu \partial^\mu \Phi_\nu$$

- Add a scalar deformation

$$\delta S = \int \lambda (\Phi_\mu \Phi^\mu)^2$$

- Can we find non-trivial fixed point?
- Yes, but just Wilson-Fisher fixed point (with twist that can be undone)
- Now with twisted scalar deformation

$$S = \int d^d x \partial_\mu \Phi^\nu \partial^\mu \Phi_\nu + \lambda (\Phi_\mu \Phi^\mu)^2 + \xi (\partial^\mu \Phi_\mu)^2$$

- Any scale w/o conformal fixed point?

QFT construction 2

- Now with twisted scalar deformation

$$S = \int d^d x \partial_\mu \Phi^\nu \partial^\mu \Phi_\nu + \lambda(\Phi_\mu \Phi^\mu)^2 + \xi(\partial^\mu \Phi_\mu)^2$$

- Any scale w/o conformal fixed point? (only at particular ξ , it is conformal)

- Do perturbative epsilon expansion
- Aharony-Fisher (1973) did the analysis and found the fixed point with $\xi = \infty$ ("Landau gauge" fixed point)
- $\lambda = 0$: free scale without conformal
- $\lambda \neq 0$: interacting scale without conformal!

Physical realization?

- Is topological twist just a mathematical tool?
- Consider Heisenberg O(3) model

$$S = \int d^3x \partial_\mu \Phi^I \partial_\mu \Phi_I + \lambda(\Phi_I \Phi^I)^2$$

- Index I was is originally space-time vector!
- Separation of spin and orbit is a (topological) twist!

- Dipolar-Dipolar interaction breaks the spin-orbit separation

$$\delta S = \int d^3x d^3y \Phi^\mu(x) \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu} \frac{1}{|x-y|} \Phi^\nu(y)$$

- So in the actual magnet, we may have to introduce "topologically twisted" scalar deformations
- Experimental evidence (say in EuO) suggests Aharony-Fisher fixed point (rather than Heisenberg) can be realized in nature (Mezei)

Going back to a theory…

- Final mission: is there any reason why the virial current has dimension d-1 exactly in Aharony-Fisher fixed point?
- B-field formulation

$$S = \int d^d x (\partial_\mu \Phi^\nu \partial^\mu \Phi_\nu + B \partial_\mu \Phi^\mu + \lambda (\Phi_\mu \Phi^\mu)^2)$$

$$J_\mu = B\Phi_\mu \quad \text{virial current}$$

- This has the shift symmetry of B generated by Φ_μ
- Ward-identity

$$\langle \partial^\mu \Phi_\mu(x) B \Phi_\nu(y) \Phi_\rho(z) \rangle = \delta^d(x-y) \langle \Phi_\nu(y) \Phi_\rho(z) \rangle$$

- $J_\mu = B\Phi_\mu$ must have dimension d-1 exactly
- Slava got satisfied, but I have a mixed feeling;)

Summary

- (Topologically) twisted RG is not explored very much
- New fixed points? SUGRA solution? Localization? (Caveat: intrinsically Euclidean)

- Applications to scale vs conformal
- I don’t buy genericity argument. There should be many scale w/o conformal fixed points.

- But we typically come up with non-trivial reasons the virial current is not renormalized… Debates going on...

AF fixed point violates bootstrap bound