Localization in supergravity and precision tests for AdS/CFT

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based on recent and current works with

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Black hole entropy

- Our starting point and motivation: black holes.
- Black holes are a theorist's laboratory to understand gravity.
  Key property:

$$\mathcal{S} = \frac{k_B\, c^3}{\mathcal{G}_N\, \hbar}\, \frac{A_H}{4} + N_0 \log A_H + \ldots + e^{-A_H} + \ldots$$

- Semi-classical physics gives the leading term in a large area expansion.

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  [Hawking'71],[Bekenstein'73]

- Corrections to Bekenstein-Hawking probe the quantum gravity regime.
- A natural question: can we give a Boltzmann interpretation of the exact entropy $\mathcal{S}$ in terms of microscopic degeneracies?

$$\mathcal{S} \stackrel{?}{=} k_B \log d$$

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- Asymptotically flat BHs: string theory successfully accounts for Bekenstein-Hawking entropy by realizing the black hole as a brane system.
  [Strominger,Vafa'96]

- In fact, brane picture is very powerful: also allows for the computation of sub-leading corrections to the entropy.
  [Maldacena,Strominger,Witten'97],[Dijkgraaf,Verlinde,Verlinde'97]
  [Maldacena,Moore,Strominger'99]

- For certain supersymmetric black holes, microscopic degeneracies are fully known as functions of the charges carried by the brane system/BH.

Thermodynamic *vs.* Boltzmann entropy (cont.)

- Asymptotically AdS BHs: recent progress for AdS_4 spherically symmetric BPS black holes via microstate counting in the dual field theory.

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- The sub-leading contributions are encoded in the matrix model.

- Followed (a lot of) generalizations to other models, other dimensions, etc...

  [Assel,Azzurli,Bobev,Cabo-Bizet,Choi,Crichigno,Hosseini,Hwang,Kim,Kim, Liu,Martelli,Murthy,Milan,Min,Nahmgoong,Nedelin,Giraldo-Rivera, Pando Zayas,Passias,Pilch,Rathee,Yokoyama,Zhao...]

Aim of this talk: a bulk perspective on the TTI

- Given the degeneracies computed in the microscopic theory (CFT), can we compute the corrections to $\mathcal{S}$ directly in the macroscopic theory (AdS)?

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- To answer, use
  - The quantum entropy of asymptotically AdS black holes to define $\mathcal{S}_{\mathsf{AdS}}$
  - Localization in the bulk gauged supergravity theory to compute $\mathcal{S}_{\mathsf{AdS}}$

[Dabholkar,Drukker,Gomes'14],[Nian,Zhang'17]

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- Then compare the bulk result for $\mathcal{S}_{\text{AdS}}$ to the TTI that gives d_{CFT}.
- Could lead to non-trivial identities akin to the ones derived in rigid SUSY theories when comparing partition functions.

Outline

1. Introduction and motivation
2. Localization in supergravity
3. Applications to AdS black hole entropy
4. Conclusion

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- It generically preserves 2 out of 8 supercharges (i.e. 1/4-BPS).
  In the near-horizon region, enhanced to 4 out of 8 supercharges (1/2-BPS).
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  [Cacciatori,Klemm'09],[Dall'Agata,Gnecchi'10],[Hristov,Vandoren'10]
- Focus on the entropy contribution from the near-horizon.
- Allows to consider a large class of asymptotically AdS BHs at once, namely all those with near-horizon attractor geometry $\mathrm{AdS}_2 \times S^2$.

- The AdS_2 factor is a key ingredient to explore the quantum entropy of BHs.
- Proposal: compute the following Euclidean expectation value [Sen'08]

The quantum entropy

$$e^{\mathcal{S}_{\mathrm{AdS}}} = \left\langle \exp\left[q_I \oint W^I_\tau \, d\tau \right] \right\rangle^{\text{finite}}_{\mathrm{AdS}_2}$$

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- 'finite' denotes a regularization procedure, due to non-compactness of AdS_2.
- In a suitable large charge limit, this is expected to reproduce the Bekenstein-Hawking entropy of the black hole.
- Holographically, corresponds to a Witten index in the dual CFT_1 counting the number of ground states (microstates).

- Localization gives an exact one-loop evaluation of path integrals.

[Witten'88],[Pestun'07],...

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- Localization relies on a fermionic symmetry operator δ_{eq} with algebra

$$\delta_{\text{eq}}{}^2 = \delta_{\mathring{\xi}}$$

  where $\delta_{\mathring{\xi}}$ generates a compact symmetry (space-time and/or internal).

- Very powerful: reduces the infinite-dimensional path integral to an integral over δ_{eq}-invariant configurations.

  Appropriate choice of symmetry $\delta_{\mathring{\xi}}$ constrains the fields to fluctuate along a restricted set of directions both in space-time and in field space.

- Used to great effect in rigid SUSY theories where δ_{eq} is a supercharge generated by a background Killing spinor associated with the fixed background geometry one wants to consider. [Festuccia,Seiberg'11]

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- How do we define δ_{eq} and $\delta_{\mathring{\xi}}$ in locally SUSY (i.e. SUGRA) theories?

- In SUGRA, SUSY & space-time translations are part of the gauge algebra.
- To make sense of δ_{eq} and $\delta_{\mathring{\xi}}$, consider space-time with a boundary.
  Prime example: AdS/CFT.
- Perform the functional integral over field fluctuations around a fixed background configuration that satisfies the boundary conditions.
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- Concretely: set up a nilpotent global BRST operator acting on both background ("classical") and quantum fields.
- Then, deform the BRST operator to an equivariant symmetry δ_{eq} by appropriately freezing the background fields and BRST ghosts.

- Gauge transformations with space-time dep. parameters $\xi^\alpha(x)$,

$$\delta\phi^i = R(\phi)^i{}_\alpha\,\xi^\alpha$$

- We assume off-shell closure,

$$\delta(\xi_1)\delta(\xi_2) - \delta(\xi_2)\delta(\xi_1) = \delta(\xi_3)\,, \quad {\xi_3}^\alpha = f_{\beta\gamma}{}^\alpha\,{\xi_1}^\beta\,{\xi_2}^\gamma$$

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- Introducing ghost fields c^α, the global BRST transformations are

$$\delta_{\text{brst}}\phi^i = R(\phi)^i{}_\alpha\,\Lambda\,c^\alpha\,, \quad \delta_{\text{brst}}c^\alpha = \tfrac{1}{2}\,f_{\beta\gamma}{}^\alpha\,c^\beta\,\Lambda\,c^\gamma$$

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- To gauge-fix the theory, also introduce a trivial doublet under BRST

$$\delta_{\text{brst}} b_\alpha = \Lambda \, B_\alpha\,, \quad \delta_{\text{brst}} B_\alpha = 0$$

- Make a background field split

$$\phi^i = \mathring{\phi}^i + \widetilde{\phi}^i$$

and introduce two corresponding sets of ghosts $\mathring{c}^\alpha$ and c^α.

- The BRST operator acts on the fields as

$$\delta_{\text{brst}}\mathring{\phi}^i = R(\mathring{\phi})^i{}_\alpha \wedge \mathring{c}^\alpha\,, \qquad \delta_{\text{brst}}\widetilde{\phi}^i = R(\phi)^i{}_\alpha \wedge (\mathring{c}+c)^\alpha - R(\mathring{\phi})^i{}_\alpha \wedge \mathring{c}^\alpha$$

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- The doublet (b_α, B_α) has the same BRST transformations as before, seen as purely quantum fields.

- The background fields $\mathring{\phi}^i$ carry the boundary values of the original fields. They are smoothly continued into the bulk.
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- The background fields are invariant under a subgroup of the background transformations, the boundary isometry group.
- In continuing into the bulk, keep the isometry group manifest: restrict the background ghosts $\mathring{c}^\alpha$ to take their values in the isometry algebra

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- Bckgd ghosts play the role of symmetry parameters for bckgd transfos. They vanish except the ones parameterizing the isometry subgroup.
- Special field representation of the BRST algebra where $\mathring{\phi}^i$ is BRST-invariant.

- Now deform the BRST algebra by imposing that the background ghosts are also BRST-invariant.
- Produces new global transformation δ_{eq}. The background is invariant,

$$\delta_{\text{eq}}\mathring{\phi}^i = 0\,, \qquad \delta_{\text{eq}}\mathring{c}^\alpha = 0$$

and the quantum fields transform as

$$\begin{aligned}\delta_{\text{eq}}\widetilde{\phi}^i &= R(\phi)^i{}_\alpha\,\Lambda\,(\mathring{c}+c)^\alpha \\ \delta_{\text{eq}}c^\alpha &= \tfrac{1}{2}\,f(\phi)_{\beta\gamma}{}^\alpha\,(\mathring{c}+c)^\beta\,\Lambda\,(\mathring{c}+c)^\gamma - \tfrac{1}{2}\,f(\mathring{\phi})_{\beta\gamma}{}^\alpha\,\mathring{c}^\beta\,\Lambda\,\mathring{c}^\gamma\end{aligned}$$

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- The algebra is no longer nilpotent, it is equivariant

From BRST to equivariant

$${\delta_{\text{eq}}}^2 = \delta_{\mathring{\xi}}\,, \qquad \left[\delta_{\text{eq}},\, \delta_{\mathring{\xi}}\right] = 0$$

- The r.h.s. is a bckgd transfo parameterized by $\mathring{\xi}^\alpha := f(\mathring{\phi})_{\beta\gamma}{}^\alpha\, \mathring{c}^\beta\, \mathring{c}^\gamma$.

- We can construct a quantum (including gauge-fixing) action

$$S_{\text{eq}}[\mathring{\phi}, \widetilde{\phi}] = \int \mathrm{d}^n x \left[\mathcal{L}_{\text{class}}(\mathring{\phi} + \widetilde{\phi}) + \delta_{\text{eq}}\left[b_\alpha F(\mathring{\phi}, \widetilde{\phi})^\alpha\right]\right]$$

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- The partition function can be written in terms of S_{eq}

$$Z[\mathring{\phi}] = \int \mathcal{D}\widetilde{\phi}^i\,\mathcal{D}c^\alpha\,\mathcal{D}b_\alpha\,\mathcal{D}B_\alpha\,\exp S_{\text{eq}}[\mathring{\phi},\widetilde{\phi}]$$

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- It is δ_{eq}-invariant, and independent of the gauge-fixing terms $F(\mathring{\phi}, \widetilde{\phi})^\alpha$.
- It depends on the boundary conditions, i.e. the background values $\mathring{\phi}^i$.
- Thanks to the equivariance of δ_{eq}, this formalism is suited for applying localization to compute $Z[\mathring{\phi}]$, even when SUSY is local.

- Deform the action $S(0) = S_{\text{eq}} \to S(\lambda) = S(0) + \lambda\, \delta_{\text{eq}} \mathcal{V}$ with $\delta_{\mathring{\xi}} \mathcal{V} = 0$.
- The deformed path integral is independent of the parameter λ.
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  Taking $\lambda \to \infty$, $Z[\mathring{\phi}]$ localizes to the critical points of the deformation.
- Using the usual fermionic functional

$$\mathcal{V} = \int \mathrm{d}^n x \sqrt{\mathring{g}} \sum_{\psi^i \in \widetilde{\phi^i}} \overline{\psi^i}\, \delta_{\text{eq}} \psi^i$$

we have the exact result

Equivariant localization in the presence of a boundary

$$Z[\mathring{\phi}] = \int_{\mathcal{M} = \{t_a | \delta_{\text{eq}} \psi^i(t_a) = 0\}} \mu(t)\, \mathrm{d}t_a \, \exp\left[S_{\text{class}}(\mathring{\phi}\,; t_a)\right] Z_{\text{1-loop}}(\mathring{\phi}\,; t_a)$$

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- $\mathcal{M}$ is characterized by the bosonic fields and the ghosts of fermionic gauge transformations, subject to $\delta_{\text{eq}} \psi^i = 0$ and the gauge-fixing $F(\mathring{\phi}, \widetilde{\phi}) = 0$.

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- $\mathcal{M}$ is characterized by the bosonic fields and the ghosts of fermionic gauge transformations, subject to $\delta_{\text{eq}} \psi^i = 0$ and the gauge-fixing $F(\mathring{\phi}, \widetilde{\phi}) = 0$.
- Note: there can also be contributions from singular field configurations.

Outline

- Apply localization to compute $\mathcal{S}_{\text{AdS}}$ in 4d $\mathcal{N} = 2$ gSUGRA.

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- Genuine Euclidean formulation available. [de Wit,VR'17]
- Field content: the Weyl multiplet (contains the metric), n_v U(1) vector multiplets (contain the gauge fields under which BH solution is charged). Also 1 vector and 1 hypermultiplet serving as gauge compensators.
- Specify the near-horizon 1/2-BPS field configuration.
- Look for off-shell fluctuations around this background satisfying $\delta_{\mathrm{eq}}\psi = 0$ and the attractor boundary conditions.

- The Weyl multiplet:

$$\mathbb{W} = (e_\mu{}^a,\, \psi_\mu{}^i,\, b_\mu,\, A_\mu,\, \mathcal{V}_\mu{}^i{}_j \mid T_{ab},\, \chi^i,\, D)$$

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- The (on-shell) compensating hypermultiplet:

$$\mathbb{H} = (A_i{}^\alpha,\, \zeta^\alpha)$$

- The gauging is specified by FI parameters ξ_I and generators $t^\alpha{}_\beta$,

$$\mathcal{D}_\mu A_i{}^\alpha = (\partial_\mu - b_\mu) A_i{}^\alpha + \tfrac{1}{2}\, \mathcal{V}_\mu{}_i{}^j\, A_j{}^\alpha - \xi_I\, W_\mu^I\, t^\alpha{}_\beta\, A_i{}^\beta$$

- In the gauge-fixed Poincaré theory, the gravitini are electrically charged.

- Weyl multiplet (bosonic) attractor configuration $\mathring{\mathbb{W}}$:

$$\mathring{g}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = v_1\Big[(r^2-1)\,\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{r^2-1}\Big] + v_2\,\mathrm{d}\Omega_2{}^2$$

$$\mathring{T}^{\mp}_{12} = \pm\frac{2}{\sqrt{v_1}} \qquad \mathring{D} = -\frac{1}{6}\Big(\frac{1}{v_1}+\frac{2}{v_2}\Big) \qquad \mathring{A}_\mu = \mathring{b}_\mu = 0$$

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- $\mathrm{SU}(2)_\mathrm{R}$ gauge field in the Weyl multiplet given in terms of the vectors,

$$\mathring{\mathcal{V}}_\mu{}^i{}_j = -2\, t^i{}_j\, \xi_I\, \mathring{W}^I_\mu$$

- Now look for off-shell fluctuations around the attractor background, i.e. solve $\delta_{\text{eq}}\psi^i_\mu = \delta_{\text{eq}}\chi^i = 0$ for arbitrary metric and Killing spinor satisfying the attractor boundary conditions. Hard problem.

- In ungauged SUGRA, this was solved for fluctuations around the full-BPS attractor configuration where, in particular, $\mathring{\mathcal{V}}_\mu{}^i{}_j = 0$.
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- The (commuting) attractor KS $\mathring{\epsilon}^i$ are precisely the background ghosts $\mathring{c}^\alpha$.

Algebra of the localizing supercharge

$$\delta_{\text{eq}}{}^2 = \mathcal{L}_\tau + \delta_{\text{SU(2)}}\Big(\frac{1}{\sqrt{v_1}}\, t^i{}_j\Big) + \delta_{\text{gauge}}(X_+, X_-)$$

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- We use the attractor geometry and background Killing spinors to look for off-shell fluctuations of gauge and matter fields.

- For vector multiplets, analyze the vanishing of gaugini variations $\delta_{\text{eq}}\Omega^{i\,I} = 0$.
- Supercharge δ_{eq} parametrized by an attractor Killing spinor. Due to the non-trivial $\text{SU}(2)_{\text{R}}$ gauge field, the gauging effects a twist on the S^2.
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- Imposing boundary conditions and smoothness, we find for each multiplet

$$X_{\pm} = \mathring{X}_{\pm} + \sum_{k=1}^{\infty} \frac{C_k^{\pm}(\theta, \varphi)}{r^k}$$

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  Fluctuations of gauge and auxiliary fields also controlled by $C_k^\pm(\theta,\varphi)$.
- Additional constraint: a specific linear combination of the fluctuation parameters is independent of the angular coordinates

$$\mathcal{M}^{\text{vec}} = \left\{ C_k^+(\theta,\varphi)^I, C_k^-(\theta,\varphi)^I \right\} = \left\{ \phi_0^I, \phi_\perp^I(\theta,\varphi) \right\}$$

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- Important: action on $\mathcal{M}$ only depends on the constant parameter ϕ_0^I.
- Add the Wilson line contribution and appropriate boundary terms to renormalize according to the 'finite' prescription.
- Final form:

$$S_{\text{ren}}(\phi_0) = p^I \, \mathcal{F}_I^+(\phi_0) + q_I \, \phi_0^I$$

- Since the $\phi_\perp^I$ are zero-modes of the renormalized action, they can only enter the exact evaluation of $\mathcal{S}_{\text{AdS}}$ via the one-loop determinant.
- The determinant receives contributions from both ϕ_0- and $\phi_\perp$-modes. Still contains a functional integral since $\phi_\perp^I(\theta, \varphi)$ are functions on S^2.
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Quantum entropy for 1/4-BPS BHs in $\mathcal{N} = 2$ gSUGRA

$$e^{\mathcal{S}_{\rm AdS}} = \int \prod_I [{\rm d}\phi_0^I]\, \delta(\xi_I \phi_0^I - 2\pi)\, \exp\big[\, p^I\, \mathcal{F}_I^+(\phi_0) + q_I\, \phi_0^I \,\big]\, Z_0(\phi_0)\, Z_\perp(\phi_0)$$

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- The one-loop determinant and the explicit form of Z_0 and $Z_\perp$ are currently under careful investigation. [Hristov,Lodato,VR]

- Comparison: M-theory on $\mathrm{AdS}_4 \times S^7$ has dual holographic description as ABJM theory (with CS level $k = 1$).
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$$d_{\mathrm{CFT}} = \int \prod_a \mathrm{d}\Delta_a \, \delta\big(\textstyle\sum_a \Delta_a - 2\pi\big) \, Z(p, \Delta) \, \exp[-q_a \, \Delta_a]$$

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- Beyond large N requires knowledge of Z_0 and $Z_\perp$.

Outline

Conclusions

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- Applicable to a wide variety of cases, including supergravity.
- In gauged supergravity, the formalism lends itself to evaluating the quantum entropy of asymptotically AdS BHs.
- Additional (technical) complications due to gauging. Still, concrete formulas are starting to emerge.
- Paves the way for precision tests of AdS/CFT correspondence beyond large N.

Some future directions

- Main priority: completing the one-loop determinant computation in gSUGRA.
- Will give an interesting gravitational foray into finite N corrections in the CFT side. Better understanding of the matrix model?
- But comparison must be done with care: $\mathcal{S}_{\text{AdS}}$ is near-horizon. Hair degrees of freedom for asymptotically AdS black holes?

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- Higher dimensions, black strings, ...

Thank you for your attention