Magnetically Charged AdS4 Black holes from M5-branes

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ArXiv : 1808.02797 with Nakwoo Kim (KyungHee U)
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A Magnetically charged AdS4 Black hole

Classically

$$ds^2 = -\left(\rho - \frac{1}{2\rho}\right)^2 dt^2 + \left(\rho - \frac{1}{2\rho}\right)^{-2} d\rho^2 + \rho^2 ds^2(\Sigma_g)$$

$$F = \frac{dx_1 \wedge dx_2}{{x_2}^2} \quad \text{(Magnetic flux for U(1) gauge field along Riemmann surface } \Sigma_{g>1})$$

- BPS Solution for **4D $\mathcal{N}$ =2 minimal gauged supergravity**

$$I = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2\right) + \text{(fermions)}$$

- Near horizon $\left(\rho = \frac{1}{2^{1/2}}\right)$: $AdS_2 \times \Sigma_g$,

  Asymptotically ($\rho \to \infty$) : AdS_4 with asymptotic boundary $\boldsymbol{R}_t \times \Sigma_g$

- In terms of AdS/CFT, the BH solution describes

  RG : (3D $\mathcal{N}$ =2 SCFT on $\boldsymbol{R}_t \times \Sigma_g$) ⟶ (1D SQM on $\boldsymbol{R}_t$)

  topological twisting : $\left(A^{(b.g)}\right)_R = -\omega(\Sigma_g)$

  Superconformal R-symmetry : <u>*"universal twist"*</u>

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From semiclassical analysis

[Bekenstein, Hawking]

$$S_{\rm BH} = \frac{A}{4G_4} = \frac{(g-1)\pi}{2G_4} + (\text{subleadings in } G_4)$$

If the BH solution (AdS4 supergravity) can be embedded into an UV complete Quantum Gravity,

We may give a non-perturbative definition of $d_{micro}(\#\ of\ micorstates\ of\ BH)$, which should satisfy

1) $d_{micro}\,(g, G_4)$ is an non-negative **integer** (after including all corrections)

2) $S_{\rm BH} = \log d_{micro}\,(g, G_4) = \frac{(g-1)\pi}{2G_4} + (\text{subleadings in } G_4).$

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In this talk,

- Embedding the BH into **M-theory** on AdS4 x M x S4 (M : hyperbolic 3-manifold)
- $d_{micro}\,(g, G_4)$ using **AdS4/CFT3 and 3d-3d relation ,** $\sum_\alpha (\text{Analytic torsion})^{g-1}$
- Check of 1) integrality at finite N $\sim (G_4)^{-1/3}$

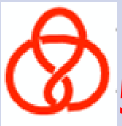
  2) Bekenstein-Hawking + sub-leadings in large N

2) $S_{\text{BH}} = \log d_{micro}\,(g, G_4) = \frac{(g-1)\pi}{2G_4} + (\text{subleadings in } G_4)$.

A Magnetically charged AdS4 BH in M-theory

BH solution with asymptotically AdS4 ⟹ Can be studied using AdS4/CFT3

Two classes of well-established AdS4/CFT3 using M-theory

AdS4/CFT3 from M2-branes	AdS4/CFT3 from M5-branes
$R^{1,2}$ × Cone(Y_7) (Y_7 : Sasakian 7-manifold) with N M2-branes on $R^{1,2}$ ⟶ $\boldsymbol{T_N[Y_7]}$ 3D $\boldsymbol{\mathcal{N}}$=2 SCFT with global $U(1)_R \subset G$ = ISO(Y_7)	$R^{1,2}$x (T^*M)x R^2 (T^*M_3: cotangent-bundle of 3-manifold M_3) with N M5 branes on $R^{1,2}$x M_3 ⟶ $\boldsymbol{T_N[M_3]}$ 3D $\boldsymbol{\mathcal{N}}$=2 SCFT, with global $U(1)_R$
M-theory on AdS4xY_7 ↓ ($G_4 = \sqrt{\frac{27}{8N^3\pi^4}\mathrm{Vol}(Y_7)}$) 4D $\boldsymbol{\mathcal{N}}$=2 Gauged supergravity with G = ISO(Y_7) $S_{\rm BH} = \frac{(g-1)\pi}{2G_4} = (g-1)\sqrt{\frac{2}{27\mathrm{Vol}(Y_7)}}N^{3/2}\pi^3$	M-Theory on Warped AdS4xM_3xS4 (for hyperbolic M_3) [Pernici ;'85] [Gauntlet-Kim-Waldram;00] ↓ ($G_4 = \frac{3\pi^2}{2N^3 vol(M)}$) 4D $\boldsymbol{\mathcal{N}}$ =2 Gauged supergravity with G =U(1) $S_{\rm BH} = \frac{(g-1)\pi}{2G_4} = \frac{(g-1)vol(M)}{3\pi}N^3$
Field theoretic description of $T_N[Y_7]$ [ABJM;08][HLLLP;08]........ e.g) $T_N[S^7/Z_k]$=ABJM model	Field theoretic description of $T_N[M_3]$ [Dimoft-Gukov-Gaiotto;11][DG-Yonekura;18] e.g) $T_{N=2}$[ 5]= (U(1) + Φ with k=-7/2)

Non-perturbative definition of d_{micro} using AdS4/CFT3

Question : *Which quantity in CFT3 corresponds to the* d_{micro} *of the BH ?*

Hints:

BH : Asymptotic AdS_4 with $\partial(\text{AdS}_4) = \boldsymbol{R}_t \times \Sigma_g$ ⟶ Near horizon $\text{AdS}_2 \times \Sigma_g$,

RG : (3D $\mathcal{N}=2$ SCFT on $\boldsymbol{R}_t \times \Sigma_g$) ⟶ (1D SQM on $\boldsymbol{R}_t$)

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Natural Answer : *the number of ground states of 3d SCFT on* Σ_g

d_{micro}

= # of supersymmetric ground states of (3D $\mathcal{N}$ =2 SCFT $on\ \Sigma_g$)

$cf)\ d^{\text{SUSY}}_{\text{micro}} := \text{Tr}_{\mathcal{H}^{E=0}(\text{3D } \mathcal{N} = 2 \text{ SCFT } on\ \Sigma_g)} (-1)^R = \text{Tr}_{\mathcal{H}(\text{3D } \mathcal{N} = 2 \text{ SCFT } on\ \Sigma_g)} (-1)^R e^{-\beta E}$

Twisted index

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Twisted index

Recently people found that [Benini-Zaffaroni ;'16] [Hosseini-Zaffaroni ;'16].......

$$\text{Log}(d^{\text{SUSY}}_{\text{micro}}(T_N[Y_7],g)) \xrightarrow{N \to \infty} \frac{(g-1)\pi}{2G_4} = (g-1)\sqrt{\frac{2}{27\text{Vol}(Y_7)}} N^{3/2}\pi^3 + \text{sub-leadings}$$

$$d^{\text{SUSY}}_{\text{micro}}(T_N[Y_7]) = d_{\text{micro}}(T_N[Y_7])\ ?? \qquad (\ Z^{\text{thermal}}(AdS_4 \times Y_7) = Z^{\text{SUSY}}(AdS_4 \times Y_7)\ ??\ )$$

Twisted index $d^{\text{SUSY}}_{\text{micro}}(g) = \text{Tr}_{H(3D\ \mathcal{N}=2\ \text{SCFT on}\ \Sigma_g)}(-1)^R e^{-\beta E}$

For $g = 1$ $(\Sigma_g = T^2)$ case : It is just usual Witten index [Kim-Kim ;'10] [Seiberg-Intrilligator ;'12]

For $g = 0$ (S^2) case [Benini-Hristov-Zaffaroni ;'15]

For general g [Benini-Zaffaroni ;'16] [Closset-Kim ;'16]

For general $3d$ $N = 2$ theory with gauge G,
the index can be written as finite sum over so called `Bethe vacua' [Nekrasov-Shatasiville] [Closset-Kim-Willet ;'17]

$$\boldsymbol{d^{\text{SUSY}}_{micro}(g) = \sum_{\alpha:\text{Bethe}-\text{vacua}} (H^{\alpha})^{g-1}},$$

Bethe vacua: solutions of eqn $\exp\left(2\pi i z_i \frac{\partial W}{\partial z_i}\right) = 1$, for $i = 1, .., \text{rank}(G)$

$W(z_1, \ldots, z_{\text{rank}(G)})$: Twisted superpotential for $2d$ $(2,2)$ theory obtained by S^1 reduction keeping all infinity KK−modes

Chiral field : $\delta W = \text{Li}_2\left(\prod z_i^{-Q_i}\right)$, CS term $\delta W = k_{ij}\text{Log}[z_i]\text{Log}[z_j]$

$H^{\alpha}(z_1, \ldots, z_{\text{rank}(G)})$: `handle gluing operator' ,

$$\text{Log}[H] = -\log \text{Det}[\partial_{\log[z_i]}\partial_{\log[z_j]}\text{Log}[W]] + \sum_{\text{Chiral}} \text{Li}_1(z_i^{-Q_i})$$

Most recent studies on AdS4 BH are about BHs from M2-branes

$$\text{Log}(d^{\text{SUSY}}_{\text{micro}}(T_N[Y_7],\text{g})) \xrightarrow{N\to\infty} \frac{(g-1)\pi}{2G_4} = (g-1)\sqrt{\frac{2}{27\text{Vol}(Y_7)}}N^{3/2}\pi^3 + \text{sub-leadings}$$

Good : Gauge theory description is simple → Matrix model technique

Flavor symmetry other than U(1) R-symmetry → Rich SUSY BHs

Bad : Flavor symmetry other than U(1) R-symmetry

→ Improperly quantized superconformal R-charge : no universal twisting

Computation of sub-leading seems to be challenging (additional Legendre transformation)

BHs from M5-branes?

$$\text{Log}(d^{\text{SUSY}}_{\text{micro}}(T_N[M_3],\text{g})) \xrightarrow{N\to\infty} (g-1)\frac{N^3\text{vol}(M_3)}{3\pi} \;??$$

Bad : UV Gauge theory description is very ugly, no matrix model ($u(1)^{N^3}$ gauge group)

Good : we can use the power of **3d-3d relation**

(***full perturbative sub-leadings*** are computable)

$d^{SUSY}_{micro}(T_N[M_3], g)$ from 3d-3d relation

3d-3d relation : ($T_N[M_3]$ on **S**) ~ (SL(N, C) Chern Simons theory on M_3) , **not duality but a relation**

[Yamazaki-Terashima ;'11][Dimoft-Gukov-Gaiotto;11]
[Yagi;13][Lee-Yamazaki;13][Cordova-Jafferis;13]

M-theoretic derivation : $6d A_{N-1}(2,0)$ theory on **S** × M_3

S = $S^3, S^1 \mathrm{x} S^2, S^3/Z_k$,....

($T_N[M_3]$ on **S**) ← $6d A_{N-1}(2,0)$ theory on **S** × M_3 → (SL(N, C) Chern Simons theory on M_3)

$d^{SUSY}_{micro}(T_N[M_3], g)$ from 3d-3d relation

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S = $S^3, S^1 x S^2, S^3/Z_k, \ldots$

Dictionary :

$T_N[M_3]$ on $R^2 \times S^1$	$SL(N,C)$ Chern Simons theory on M_3
Bethe vacuum α	SL(N,C) irreducible flat connection A^α
Handle gluing operator H^α	N $\mathrm{Exp}[-2S^\alpha(1)]$

$dA^\alpha + A^\alpha \wedge A^\alpha = 0$

$\frac{\delta CS[A]}{\delta A} = dA + A \wedge A$

$CS[A] = \int_M \mathrm{tr}\left(AdA + \frac{2}{3}A^2\right)$

Recall that $d^{SUSY}_{micro}(g) = \sum_{\alpha:\mathrm{Bethe-vacua}} (H^\alpha)^{g-1}$, $(for\ M_3\ s.that\ H_1(M_3, Z_N) = 0)$

$S^\alpha(n)$: $n\ loop\ perturbative\ expansion\ coefficient\ around\ a\ flat\ connection\ A^\alpha$

$$\log \int \frac{[d(\delta A)]}{(\mathrm{gauge})} \mathrm{Exp}\left[\frac{1}{2\hbar} CS[A^\alpha + \delta A]\right] \longrightarrow \frac{1}{\hbar} S^\alpha(0) + S^\alpha(1) + ..\hbar^{n-1} S^\alpha(n) + \ldots$$

$S^\alpha(1) = \frac{1}{4} \mathrm{Log}\left[\frac{\left(\det' \Delta_0^{(\alpha)}\right)^3}{\left(\det' \Delta_1^{(\alpha)}\right)}\right]$ (Ray-singer torsion)

$\Delta_n^{(\alpha)} = * d_A * d_A + d_A * d_A *$, $d_A = d + A^\alpha \wedge$
(Laplacian acting on n-form twisted by A^α)

$d^{SUSY}_{micro}(T_N[M_3]$,g) from 3d-3d relation

3d-3d relation : $(T_N[M_3]$ on **S**$) \sim (\mathrm{SL}(N,C)$ Chern Simons theory on $M_3)$, **not duality but a relation**

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$$CS[A] = \int_M \mathrm{tr}\left(AdA + \frac{2}{3}A^2\right)$$

Derivation : From $M = \left(\bigcup_{i=1}^{k} \Delta_i \bigcup_i^m S_i\right)/\sim$, Δ: (*ideal tetrahedron*), S: *solid torus*

[Dimofte-Garoufalidis;12]
[DG-Romo-Yamazaki;17]

Field theoretic description of $T_N[M_3]$ [Dimofte-Gaiotto-Gukov;11] [DG-Yonekura;18]

(Localization)

N $\mathrm{Exp}[-2S^\alpha(1)]$ $=$ H^α

Observation, not M-theoretic derivation

$d^{SUSY}_{micro}(T_N[M_3], g)$ from 3d-3d relation

3d-3d relation : $(T_N[M_3] \text{ on } R^2 \times S^1) \sim (SL(N,C) \text{ Chern Simons theory on } M_3)$, not duality but a relation

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$$S^\alpha(1) = \frac{1}{4}\mathrm{Log}\left[\frac{\left(\det{}'\Delta_0{}^{(\alpha)}\right)^3}{\left(\det{}'\Delta_1{}^{(\alpha)}\right)}\right] \quad \text{(Ray-singer torsion)}$$

$$d^{SUSY}_{micro}(T_N[M_3], g) = N^{1-g}\sum_\alpha \exp(-2S^\alpha(1)[M_3\,;N]\,)^{g-1}$$

We reduce the microstates counting of BH to a mathematical problem !!
Use mathematical results to study BH entropy !!

$d^{SUSY}_{micro}(T_N[M_3],g)$ from 3d-3d relation

3d-3d relation : $(T_N[M_3]$ on $R^2 \times S^1) \sim (SL(N,C)$ Chern Simons theory on $M_3)$, not duality but a relation

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We reduce the microstates counting of BH to a mathematical problem !!
Use mathematical results to study BH entropy !!

Using the expression, Let us check followings

1) $d^{SUSY}_{micro}(T_N[M_3],g)$ is an integer (after including all corrections)

2) $S_{\mathrm{BH}} = \log d^{SUSY}_{micro}(T_N[M_3],g) = \frac{(g-1)\pi}{2G_4} = \frac{(g-1)vol(M)}{3\pi}N^3 + (\text{subleadings in } 1/N).$

Integrality of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

Irreducible flat connection : $dA^{\alpha} + A^{\alpha} \wedge A^{\alpha} = 0$

$$S^{\alpha}(1)[\mathrm{M};\mathrm{N}] = \frac{1}{4}\mathrm{Log}\left[\frac{\left(\det{}'\Delta_0{}^{(\alpha)}\right)^3}{\left(\det{}'\Delta_1{}^{(\alpha)}\right)}\right] \text{ (Ray-singer torsion)}$$

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = N^{g-1}\sum_{\alpha} \exp(-2S^{\alpha}(1)[M_3\ ;N]\ )^{g-1}$$

Conjecture (?)

$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) \in \boldsymbol{Z}$

Integrality of $d^{\text{SUSY}}_{micro}(T_N[M_3],\mathrm{g})$

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$$d^{\text{SUSY}}_{micro}(T_N[M_3],\mathrm{g}) = N^{g-1}\sum_\alpha \exp(-2S^\alpha(1)[M_3\,;N]\,)^{g-1}$$

Conjecture (?)

$d^{\text{SUSY}}_{micro}(T_N[M_3],\mathrm{g}) \in \boldsymbol{Z}$

e.g) $M_3 =$  $_5$, $N=2$

[Computable using tools developed by mathematicians]

$\{\exp(-2S^\alpha(1)[M_3;N]\,)\}_{\alpha=1,2,3,4}$

={-1.90538-0.568995 i,-1.90538+0.568995 i,1.73992, 2.57085} $\left(x^4 - 1/2\,x^3 - 8x^2 + 283/16 = 0\right)$

$\Longrightarrow$ $\{d^{\text{SUSY}}_{micro}(T_N[M_3],\mathrm{g})\}_{g=0,1,2,..}$ = { 0, 4, 1, 65, 97, 1045,...}

Conjecture (?)

$d^{\text{SUSY}}_{micro}(T_N[M_3],\mathrm{g=0}) = 0$

No SUSY black hole for **g=0**

Large N of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = N^{g-1}\sum_\alpha \exp(-2S^\alpha(1)[M_3\ ;N]\,)^{g-1}$$

Two canonical irreducible flat connections A^{hyp} *and* $A^{\overline{hyp}}$

$A^{hyp} = \rho_N[\omega + ie]\ ,\quad A^{\overline{hyp}} = \rho_N[\omega - ie]$

$\rho_N: \mathrm{su}(2) \to \mathrm{su}(N), N -$ dimensional irred representation

ω: spin connection
e: vielbein

for unique hyperbolic metric on M satisfying $R_{\mu\nu} = -2g_{\mu\nu}$

Both of them can be locally considered as $so(3)$ *valued 1 forms*

$\omega \pm ie$: $sl(2,C)$ *valued 1 form satisyfing flat connection equation* $dA + A \wedge A$

These two give dominant contributions to the d^{SUSY}_{micro} in large N (g>1)

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = \left(H^{\text{hyp}}\right)^{g-1} + (c.c)$$
$$+\text{exponentially small in } 1/N$$

$$H^\alpha = N\exp(-2S^\alpha(1)[M_3\ ;N]\,)$$

Large N of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

Two canonical flat connections give dominant contributions

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = \left(H^{\text{hyp}}\right)^{g-1} + (c.c) \quad + \text{exponentially small in } 1/N$$

$$H^{\alpha} = N \exp(-2S^{\alpha}(1)[M_3\ ; N])$$

Mathematicians studied [Muller;14]……

$$2\text{Re}[S^{\text{hyp}}(1)[M_3; N]] \longrightarrow -\frac{(N^3-N)\text{vol}(M_3)}{3\pi} + \text{a}(M_3)(N-1) + \text{b}(M_3) + \text{o}(\text{e}^{-\text{N}})$$

$a(M) := a_1(M) + a_2(M)$ where

$$a_1(M) := 2\text{Re}[S^{\text{hyp}}(1)[M_3; \boldsymbol{N=1}]]$$

$$a_2(M) := -\Re\mathfrak{e} \sum_{[\gamma]} \sum_{s=1}^{\infty} \frac{1}{s} \frac{e^{-s\ell_{\mathbb{C}}(\gamma)}}{1-e^{-s\ell_{\mathbb{C}}(\gamma)}} = \sum_{[\gamma]} \sum_{m=1}^{\infty} \log|1-e^{-m\ell_{\mathbb{C}}(\gamma)}|\,,$$

$$b(M) := \Re\mathfrak{e} \sum_{[\gamma]} \sum_{s=1} \frac{1}{s} \left(\frac{e^{-s\ell_{\mathbb{C}}(\gamma)}}{1-e^{-s\ell_{\mathbb{C}}(\gamma)}} \right)^2$$

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = 2\text{Cos}[\theta_N[\boldsymbol{M_3}]]\exp\left((g-1)(\frac{N^3-N}{3\pi}\text{vol}(M) - a\,(N-1) - b + \log N + \text{o}(\text{e}^{-\text{N}})) \right) + \text{o}(\text{e}^{-\text{N}})$$

(Relative phase)

Large N of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

We finally have

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = 2\text{Cos}[\theta_N[M_3]]\exp\left((g-1)(\frac{N^3-N}{3\pi}\text{vol}(M) - a\,(N-1) - b + \log N + \text{o}(\text{e}^{-\text{N}}))\right) + \text{o}(\text{e}^{-\text{N}})$$

(Relative phase) *(Bekenstein-Hawking)* (Logarithmic correction $\frac{(1-g)}{3}\log G_4$)

[Liu, Pando Zayas, Rathee, Zhao;17]

$a(M) := a_1(M) + a_2(M)$ where

$$a_1(M) := 2\text{Re}[S^{\text{hyp}}(1)[M_3;\boldsymbol{N}=\boldsymbol{1}]$$

$$a_2(M) := -\mathfrak{Re}\sum_{[\gamma]}\sum_{s=1}^{\infty}\frac{1}{s}\frac{e^{-s\ell_{\mathbb{C}}(\gamma)}}{1-e^{-s\ell_{\mathbb{C}}(\gamma)}} = \sum_{[\gamma]}\sum_{m=1}^{\infty}\log|1-e^{-m\ell_{\mathbb{C}}(\gamma)}|\,,$$

$$b(M) := \mathfrak{Re}\sum_{[\gamma]}\sum_{s=1}\frac{1}{s}\left(\frac{e^{-s\ell_{\mathbb{C}}(\gamma)}}{1-e^{-s\ell_{\mathbb{C}}(\gamma)}}\right)^2$$

M2 branes wrapping 1-cycle in M?

Summary and future directions

We study microstate counting $d^{\text{SUSY}}_{micro}(T_N[M_3], g)$ for 4d magnetically charged BHs from wrapped N M5-branes on 3-manifold M_3

Using a 3d-3d relation, the counting to a mathematical problem

$$d^{\text{SUSY}}_{micro}(T_N[M_3], g) = N^{g-1} \sum_\alpha \exp(-2S^\alpha(1)[M_3\ ; N]\ )^{g-1}$$

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Handle gluing operator H^α	N $\text{Exp}[-2S^\alpha(1)]$

Then, using known mathematical results

$$d^{\text{SUSY}}_{micro}(T_N[M_3], g) = 2\text{Cos}[\theta_N[M_3]]\exp\left((g-1)\left(\frac{N^3-N}{3\pi}\text{vol}(M) - a\,(N-1) - b + \log N + o(e^{-N})\right)\right) + o(e^{-N})$$

(Relative phase) *(Bekenstein-Hawking)* *(Logarithmic correction* $\frac{(1-g)}{3}\log G_4$)

Future work : 1) **6D derivation** of the 3d-3d relation for twisted index

2) Perturbative corrections a, b from **quantum gravity**? (contributions from M2?)

3) Curious **integral properties of ray-singer torsions** on hyperbolic 3-manifolds